The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
www.cemc.uwaterloo.ca

Cayley Contest

(Grade 10)

Thursday, February 24, 2011

Time: 60 minutes
Calculators are permitted

Instructions

1. Do not open the Contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper left corner.
5. **Be certain that you code your name, age, sex, grade, and the Contest you are writing in the response form. Only those who do so can be counted as official contestants.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor tells you to begin, you will have *sixty* minutes of working time.

The names of some top-scoring students will be published in the PCF Results on our Web site, http://www.cemc.uwaterloo.ca.

Scoring:	There is *no penalty* for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. The value of $(5+2)+(8+6)+(4+7)+(3+2)$ is

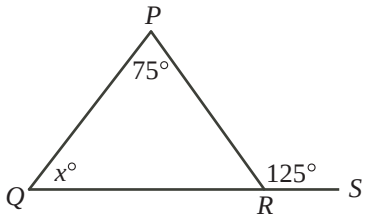
   (A) 35 **(B)** 37 **(C)** 40 **(D)** 45 **(E)** 47

2. If $(-1)(2)(x)(4) = 24$, then x equals

   (A) 4 **(B)** -3 **(C)** -1 **(D)** 2 **(E)** -4

3. In the diagram, R lies on line segment QS. What is the value of x?

   (A) 50 **(B)** 55 **(C)** 75
   (D) 100 **(E)** 105



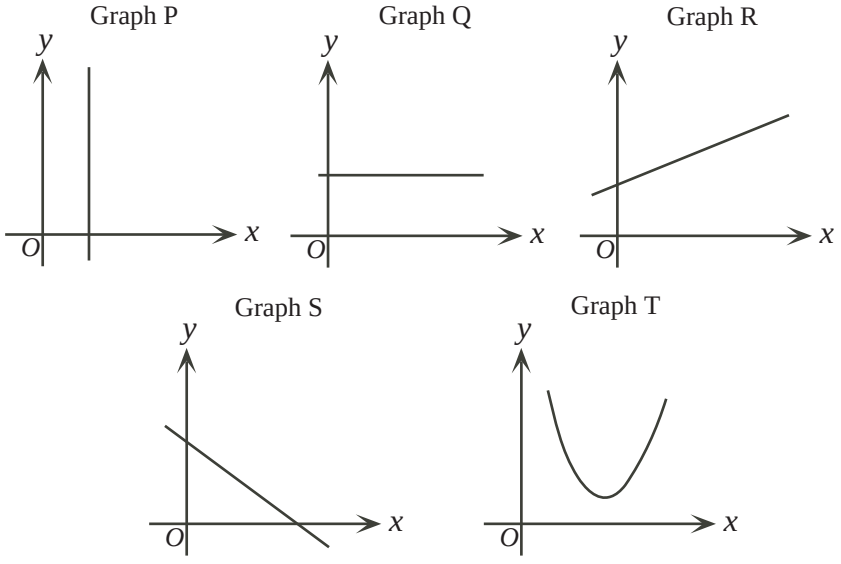
4. When a number is tripled, then decreased by 5, the result is 16. What is the original number?

   (A) 3 **(B)** 5 **(C)** 7 **(D)** 9 **(E)** 11

5. The expression $\sqrt{13+\sqrt{7+\sqrt{4}}}$ is equal to

   (A) 7 **(B)** 8 **(C)** 6 **(D)** 4 **(E)** 5

6. Which of the five graphs is linear with a slope of 0?



   (A) Graph P **(B)** Graph Q **(C)** Graph R **(D)** Graph S **(E)** Graph T

7. After a fair die with faces numbered 1 to 6 is rolled, the number on the top face is x. Which of the following is most likely?

(A) x is greater than 2 **(B)** x equals 4 or 5 **(C)** x is even
(D) x is less than 3 **(E)** x equals 3

8. If 2.4×10^8 is doubled, then the result is equal to

(A) 2.4×20^8 **(B)** 4.8×20^8 **(C)** 4.8×10^8 **(D)** 2.4×10^{16} **(E)** 4.8×10^{16}

9. A proposed new \$5 coin is called the "foonie". The foonie's two faces are identical and each has area 5 cm^2. The thickness of the foonie is 0.5 cm. How many foonies are in a stack that has a volume of 50 cm^3?

(A) 5 **(B)** 10 **(C)** 15 **(D)** 20 **(E)** 40

10. The Athenas are playing a 44 game season. Each game results in a win or a loss, and cannot end in a tie. So far, they have 20 wins and 15 losses. In order to make the playoffs, they must win at least 60% of all of their games. What is the smallest number of their remaining games that they must win to make the playoffs?

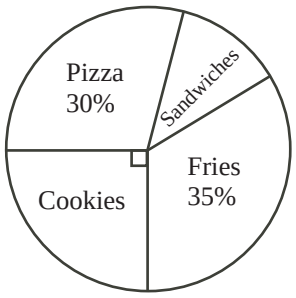
(A) 8 **(B)** 9 **(C)** 5 **(D)** 6 **(E)** 7

Part B: Each correct answer is worth 6.

11. The operation "∇" is defined by $(a, b)\nabla(c, d) = ac + bd$.
For example $(1, 2)\nabla(3, 4) = (1)(3) + (2)(4) = 11$.
The value of $(3, 1)\nabla(4, 2)$ is

(A) 10 **(B)** 11 **(C)** 13 **(D)** 14 **(E)** 24

12. The circle graph shown illustrates the results of a survey taken by the Cayley H.S. Student Council to determine the favourite cafeteria food. How many of the 200 students surveyed said that their favourite food was sandwiches?



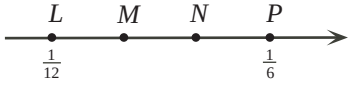
(A) 10 **(B)** 20 **(C)** 35
(D) 50 **(E)** 70

13. In the subtraction shown, K, L, M, and N are digits. What is the value of $K + L + M + N$?

$$\begin{array}{rcccc} & 5 & K & 3 & L \\ - & M & 4 & N & 1 \\ \hline & 4 & 4 & 5 & 1 \end{array}$$

(A) 20 **(B)** 19 **(C)** 16
(D) 13 **(E)** 9

14. On the number line, points M and N divide LP into three equal parts. What is the value at M?



(A) $\frac{1}{7}$ **(B)** $\frac{1}{8}$ **(C)** $\frac{1}{9}$
(D) $\frac{1}{10}$ **(E)** $\frac{1}{11}$

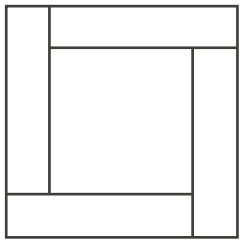
15. The points $Q(1,-1)$, $R(-1,0)$ and $S(0,1)$ are three vertices of a parallelogram. The coordinates of the fourth vertex of the parallelogram could be

(A) $(-2,2)$ **(B)** $(0,-1)$ **(C)** $(0,0)$ **(D)** $(\frac{3}{2},\frac{1}{2})$ **(E)** $(-1,1)$

16. A gumball machine that randomly dispenses one gumball at a time contains 13 red, 5 blue, 1 white, and 9 green gumballs. What is the least number of gumballs that Wally must buy to *guarantee* that he receives 3 gumballs of the same colour?

(A) 6 **(B)** 9 **(C)** 4 **(D)** 7 **(E)** 8

17. Four congruent rectangles and a square are assembled without overlapping to form a large square, as shown. Each of the rectangles has a perimeter of 40 cm. The total area of the large square is

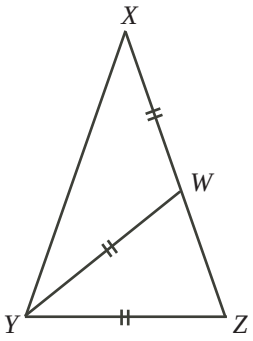


(A) 160 cm^2 **(B)** 200 cm^2 **(C)** 400 cm^2
(D) 800 cm^2 **(E)** 1600 cm^2

18. When 100 is divided by 12, the remainder is 4.
When 100 is divided by a positive integer x, the remainder is 10.
When 1000 is divided by x, the remainder is

(A) 10 **(B)** 100 **(C)** 0 **(D)** 1 **(E)** 90

19. In the diagram, $\triangle XYZ$ is isosceles with $XY = XZ$. Also, point W is on XZ so that $XW = WY = YZ$. The measure of $\angle XYW$ is



(A) 18° **(B)** 30° **(C)** 45°
(D) 36° **(E)** 60°

20. For how many positive integers n, with $n \leq 100$, is n^3+5n^2 the square of an integer?

(A) 7 **(B)** 8 **(C)** 9 **(D)** 10 **(E)** 11

Part C: Each correct answer is worth 8.

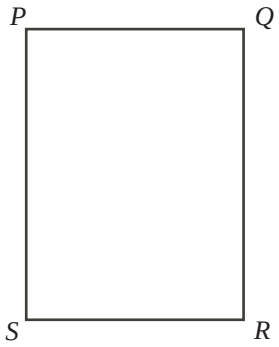
21. Suppose that x and y are positive numbers with

$$\begin{aligned} xy &= \tfrac{1}{9} \\ x(y+1) &= \tfrac{7}{9} \\ y(x+1) &= \tfrac{5}{18} \end{aligned}$$

What is the value of $(x+1)(y+1)$?

(A) $\frac{11}{6}$ **(B)** $\frac{8}{9}$ **(C)** $\frac{16}{9}$ **(D)** $\frac{10}{9}$ **(E)** $\frac{35}{18}$

22. The top section of an 8 cm by 6 cm rectangular sheet of paper is folded along a straight line so that when the top section lies flat on the bottom section, corner P lies on top of corner R. The length of the crease, in cm, is



(A) 6.25 **(B)** 7 **(C)** 7.5
(D) 7.4 **(E)** 10

23. A *Fano table* is a table with three columns where

- each entry is an integer taken from the list $1, 2, 3, \ldots, n$, and
- each row contains three different integers, and
- for each possible pair of distinct integers from the list $1, 2, 3, \ldots, n$, there is exactly one row that contains both of these integers.

1	2	4
2	3	5
3	4	6
4	5	7
5	6	1
6	7	2
7	1	3

The number of rows in the table will depend on the value of n. For example, the table shown is a Fano table with $n = 7$. (Notice that 2 and 6 appear in the same row only once, as does every other possible pair of the numbers $1, 2, 3, 4, 5, 6, 7$.) For how many values of n with $3 \leq n \leq 12$ can a Fano table be created?

(A) 2 **(B)** 3 **(C)** 5
(D) 6 **(E)** 7

24. Dolly, Molly and Polly each can walk at 6 km/h. Their one motorcycle, which travels at 90 km/h, can accommodate at most two of them at once (and cannot drive by itself!). Let t hours be the time taken for all three of them to reach a point 135 km away. Ignoring the time required to start, stop or change directions, what is true about the smallest possible value of t?

(A) $t < 3.9$ **(B)** $3.9 \leq t < 4.1$ **(C)** $4.1 \leq t < 4.3$
(D) $4.3 \leq t < 4.5$ **(E)** $t \geq 4.5$

25. Two numbers a and b with $0 \leq a \leq 1$ and $0 \leq b \leq 1$ are chosen at random. The number c is defined by $c = 2a + 2b$. The numbers a, b and c are each rounded to the nearest integer to give A, B and C, respectively. (For example, if $a = 0.432$ and $b = 0.5$, then $c = 1.864$, and so $A = 0$, $B = 1$, and $C = 2$.) What is the probability that $2A + 2B = C$?

(A) $\frac{15}{32}$ **(B)** $\frac{3}{8}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{7}{16}$ **(E)** $\frac{3}{4}$

The CENTRE for EDUCATION in MATHEMATICS and COMPUTING

For students...

Thank you for writing the 2011 Cayley Contest!
In 2010, more than 81 000 students around the world registered to write the Pascal, Cayley and Fermat Contests.

Encourage your teacher to register you for the Galois Contest which will be written on April 13, 2011.

Visit our website to find

- More information about the Galois Contest
- Free copies of past contests
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For teachers...

Visit our website to

- Register your students for the Fryer, Galois and Hypatia Contests which will be written on April 13, 2011
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www.cemc.uwaterloo.ca

Canadian Mathematics Competition

An activity of the Centre for Education in Mathematics and Computing, University of Waterloo, Waterloo, Ontario

Cayley Contest (Grade 10)

Thursday, February 25, 2010

Time: 60 minutes
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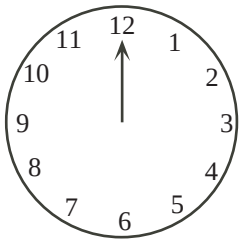
Part A: Each correct answer is worth 5.

1. The value of $6 + 4 \div 2$ is

   (A) 5 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 9

2. The minute hand on a clock points at the 12. The minute hand then rotates 120° clockwise. Which number will it be pointing at?

   (A) 6 **(B)** 2 **(C)** 4 **(D)** 3 **(E)** 5



3. If $x + \sqrt{25} = \sqrt{36}$, then x equals

   (A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 11

4. When simplified, $\dfrac{1}{2 + \frac{2}{3}}$ is equal to

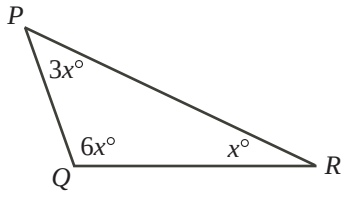
   (A) $\frac{1}{8}$ **(B)** $\frac{5}{2}$ **(C)** $\frac{5}{8}$ **(D)** $\frac{1}{2}$ **(E)** $\frac{3}{8}$

5. A rectangle has a length of $\frac{3}{5}$ and an area of $\frac{1}{3}$. What is the width of the rectangle?

   (A) $\frac{1}{5}$ **(B)** $\frac{5}{9}$ **(C)** $\frac{14}{15}$ **(D)** $\frac{15}{14}$ **(E)** $\frac{9}{5}$

6. What is the measure of the largest angle in $\triangle PQR$?

   (A) 144° **(B)** 96° **(C)** 120° **(D)** 60° **(E)** 108°

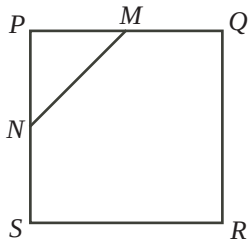


7. The mean (average) of 5 consecutive integers is 9. What is the smallest of these 5 integers?

   (A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

8. Square $PQRS$ has an area of 900. M is the midpoint of PQ and N is the midpoint of PS. What is the area of triangle PMN?

   (A) 100 **(B)** 112.5 **(C)** 150 **(D)** 225 **(E)** 180



9. Which of the following lines, when drawn together with the x-axis and the y-axis, encloses an isosceles triangle?

(A) $y = 4x + 4$ **(B)** $y = \frac{1}{2}x + 4$ **(C)** $y = -x + 4$
(D) $y = 2x + 4$ **(E)** $y = -3x + 4$

10. There are 400 students at Pascal H.S., where the ratio of boys to girls is 3 : 2. There are 600 students at Fermat C.I., where the ratio of boys to girls is 2 : 3. When considering all the students from both schools, what is the ratio of boys to girls?

(A) 2 : 3 **(B)** 12 : 13 **(C)** 1 : 1 **(D)** 6 : 5 **(E)** 3 : 2

Part B: Each correct answer is worth 6.

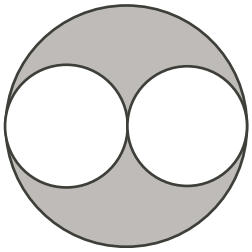
11. If x and y are positive integers with $x + y = 31$, then the largest possible value of xy is

(A) 240 **(B)** 238 **(C)** 255 **(D)** 248 **(E)** 242

12. The price of each item at the Gauss Gadget Store has been reduced by 20% from its original price. An MP3 player has a sale price of \$112. What would the same MP3 player sell for if it was on sale for 30% off of its original price?

(A) \$78.40 **(B)** \$100.80 **(C)** \$89.60 **(D)** \$168.00 **(E)** \$98.00

13. In the diagram, the smaller circles touch the larger circle and touch each other at the centre of the larger circle. The radius of the larger circle is 6. What is the area of the shaded region?



(A) 27π **(B)** 6π **(C)** 9π
(D) 18π **(E)** 36π

14. How many ordered pairs (a, b) of positive integers satisfy $a^2 + b^2 = 50$?

(A) 0 **(B)** 1 **(C)** 3 **(D)** 5 **(E)** 7

15. A loonie is a \$1 coin and a dime is a \$0.10 coin. One loonie has the same mass as 4 dimes. A bag of dimes has the same mass as a bag of loonies. The coins in the bag of loonies are worth \$400 in total. How much are the coins in the bag of dimes worth?

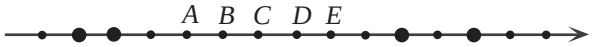
(A) \$40 **(B)** \$100 **(C)** \$160 **(D)** \$1000 **(E)** \$1600

16. The odd numbers from 5 to 21 are used to build a 3 by 3 magic square. (In a magic square, the numbers in each row, the numbers in each column, and the numbers on each diagonal have the same sum.) If 5, 9 and 17 are placed as shown, what is the value of x?

	5	
9		17
x		

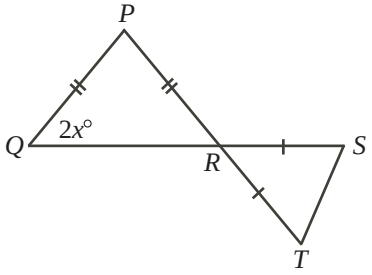
(A) 7 **(B)** 11 **(C)** 13
(D) 15 **(E)** 19

17. In the diagram, the number line is marked at consecutive integers, but the numbers themselves are not shown. The four larger dots represent two numbers that are multiples of 3 and two numbers that are multiples of 5. Which point represents a number which is a multiple of 15?



(A) A **(B)** B **(C)** C **(D)** D **(E)** E

18. In the diagram, R is the point of intersection of PT and QS, $PQ = PR$, and $RS = RT$. If $\angle PQR = 2x^\circ$, then the measure of $\angle RST$, in degrees, is



(A) $45 - x$ **(B)** $90 + \frac{1}{2}x$ **(C)** $90 - \frac{1}{2}x$
(D) $45 + 2x$ **(E)** $90 - x$

19. How many 3-digit positive integers have exactly one even digit?

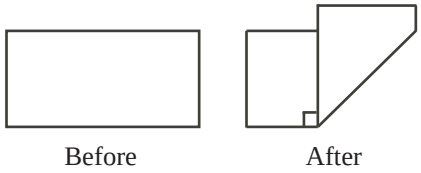
(A) 350 **(B)** 450 **(C)** 375 **(D)** 75 **(E)** 125

20. What is the largest positive integer n that satisfies $n^{200} < 3^{500}$?

(A) 13 **(B)** 14 **(C)** 15 **(D)** 16 **(E)** 17

Part C: Each correct answer is worth 8.

21. A rectangular piece of paper measures 17 cm by 8 cm. It is folded so that a right angle is formed between the two segments of the original bottom edge, as shown. What is the area of the new figure?



(A) 104 cm^2 **(B)** 81 cm^2 **(C)** 72 cm^2 **(D)** 168 cm^2 **(E)** 64 cm^2

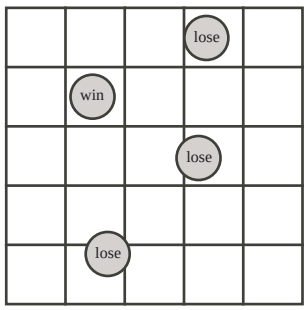
22. A sequence consists of 2010 terms. Each term after the first is 1 larger than the previous term. The sum of the 2010 terms is 5307. When every second term is added up, starting with the first term and ending with the second last term, the sum is

(A) 2155 **(B)** 2153 **(C)** 2151 **(D)** 2149 **(E)** 2147

23. Connie has a number of gold bars, all of different weights. She gives the 24 lightest bars, which weigh 45% of the total weight, to Brennan. She gives the 13 heaviest bars, which weigh 26% of the total weight, to Maya. She gives the rest of the bars to Blair. How many bars did Blair receive?

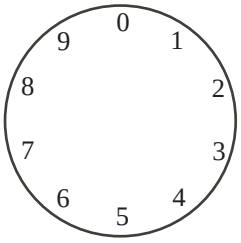
(A) 14 **(B)** 15 **(C)** 16 **(D)** 17 **(E)** 18

24. A coin that is 8 cm in diameter is tossed onto a 5 by 5 grid of squares each having side length 10 cm. A coin is in a winning position if no part of it touches or crosses a grid line, otherwise it is in a losing position. Given that the coin lands in a random position so that no part of it is off the grid, what is the probability that it is in a winning position?



(A) $\frac{25}{441}$ **(B)** $\frac{1}{25}$ **(C)** $\frac{1}{49}$
(D) $\frac{5}{147}$ **(E)** $\frac{4\pi}{25}$

25. Steve places a counter at 0 on the diagram. On his first move, he moves the counter 1^1 step clockwise to 1. On his second move, he moves 2^2 steps clockwise to 5. On his third move, he moves 3^3 steps clockwise to 2. He continues in this manner, moving n^n steps clockwise on his nth move. At which position will the counter be after 1234 moves?



(A) 1 **(B)** 3 **(C)** 5
(D) 7 **(E)** 9

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For students...

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Check out the CEMC's group on Facebook, called "Who is The Mathiest?".

Encourage your teacher to register you for the Galois Contest which will be written on April 9, 2010.
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An activity of the Centre for Education in Mathematics and Computing, University of Waterloo, Waterloo, Ontario

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C.M.C. Sponsors

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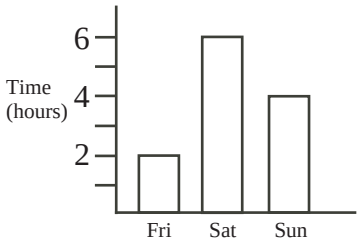
Scoring:	There is *no penalty* for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. The value of $\dfrac{10^2 - 10}{9}$ is

   (A) 10 **(B)** 1 **(C)** 7 **(D)** 2009 **(E)** 11

2. The graph shows the number of hours Deepit worked over a three day period. What is the total number of hours that he worked on Saturday and Sunday?



   (A) 2 **(B)** 4 **(C)** 6
   (D) 8 **(E)** 10

3. If $3(-2) = \nabla + 2$, then ∇ equals

   (A) -2 **(B)** 0 **(C)** -8 **(D)** -6 **(E)** -4

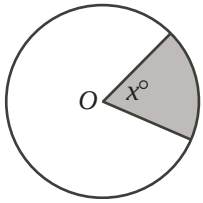
4. If $\sqrt{5+n} = 7$, the value of n is

   (A) 4 **(B)** 9 **(C)** 24 **(D)** 44 **(E)** 74

5. $3^2 + 4^2 + 12^2$ is equal to

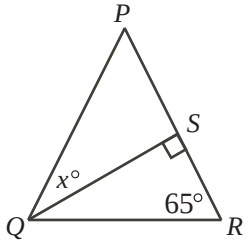
   (A) 13^2 **(B)** 19^2 **(C)** 17^2 **(D)** 15^2 **(E)** 11^2

6. In the diagram, the centre of the circle is O. The area of the shaded region is 20% of the area of the circle. The value of x is



   (A) 18 **(B)** 45 **(C)** 60
   (D) 72 **(E)** 90

7. In the diagram, $PQ = PR$ and $\angle QRP = 65°$. The value of x is



   (A) 45 **(B)** 30 **(C)** 50
   (D) 60 **(E)** 40

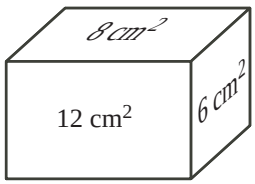
8. When three consecutive positive integers are multiplied together, the answer is always

   (A) odd **(B)** a multiple of 6 **(C)** a multiple of 12
   (D) a multiple of 4 **(E)** a multiple of 5

9. If Francis spends $\frac{1}{3}$ of his day sleeping, $\frac{1}{4}$ of his day studying and $\frac{1}{8}$ of his day eating, how many hours in the day does he have left?

   (A) 4 **(B)** 6 **(C)** 5 **(D)** 7 **(E)** 9

10. The front of a rectangular prism has an area of 12 cm^2, the side has an area of 6 cm^2, and the top has area 8 cm^2. The volume of the prism in cm^3, is



(A) 24 **(B)** 26 **(C)** 48
(D) 72 **(E)** 52

Part B: Each correct answer is worth 6.

11. Gillian has a collection of 50 songs that are each 3 minutes in length and 50 songs that are each 5 minutes in length. What is the maximum number of songs from her collection that she can play in 3 hours?

(A) 100 **(B)** 36 **(C)** 56 **(D)** 60 **(E)** 80

12. In the table shown, a sequence starts with 2 in the top left corner. Moving across each row, each box is filled with a number 3 greater than the number to its left. The leftmost number in each row is 3 greater than the greatest in the previous row. When all of the boxes are filled in, the value of x is

2	5	8	11	14	17
20	23	26	29	32	35
38					
					x

(A) 101 **(B)** 104 **(C)** 107
(D) 110 **(E)** 113

13. Filipa plays a game. She starts with a row of 15 squares and a coin on the centre square. Filipa then rolls a die. If she rolls an even number, she moves the coin that many squares to the right; if she rolls an odd number, she moves the coin that many squares to the left. If the results of six rolls were $1, 2, 3, 4, 5, 6$, where would her coin be located?

(A) On the square where it started
(B) 1 square to the right of where it started
(C) 2 squares to the right of where it started
(D) 2 squares to the left of where it started
(E) 3 squares to the right of where it started

14. A positive integer larger than 2 is called composite if it is not prime. What is the smallest prime number that is the sum of three *different* composite numbers?

(A) 11 **(B)** 13 **(C)** 17 **(D)** 19 **(E)** 23

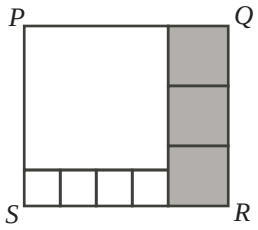
15. A list of 5 positive integers has all of the following properties:

- the only integer in the list that occurs more than once is 8,
- its median is 9, and
- its average (mean) is 10.

What is the largest possible integer that could appear in the list?
(Note: The median of a set of five positive integers is the middle integer when the set is arranged in increasing order.)

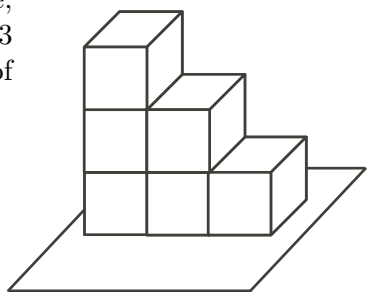
(A) 15 **(B)** 16 **(C)** 17 **(D)** 24 **(E)** 25

16. Rectangle $PQRS$ is divided into eight squares, as shown. The side length of each shaded square is 10. What is the length of the side of the largest square?



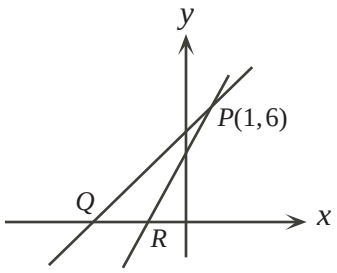
(A) 18 **(B)** 24 **(C)** 16
(D) 23 **(E)** 25

17. Six dice are stacked on the floor as shown. On each die, the 1 is opposite the 6, the 2 is opposite the 5, and the 3 is opposite the 4. What is the maximum possible sum of numbers on the 21 visible faces?



(A) 69 **(B)** 88 **(C)** 89
(D) 91 **(E)** 96

18. A line with slope equal to 1 and a line with slope equal to 2 intersect at the point $P(1,6)$, as shown. The area of $\triangle PQR$ is



(A) 6 **(B)** 9 **(C)** 12
(D) 15 **(E)** 18

19. How many integers n are there with the property that the product of the digits of n is 0, where $5000 \leq n \leq 6000$?

(A) 332 **(B)** 270 **(C)** 301 **(D)** 272 **(E)** 299

20. On Monday, Hank drove to work at an average speed of 70 km/h and arrived 1 minute late. On Tuesday, he left at the same time and took the same route. This time he drove at an average speed of 75 km/h and arrived 1 minute early. How long is his route to work?

(A) 30 km **(B)** 35 km **(C)** 45 km **(D)** 50 km **(E)** 60 km

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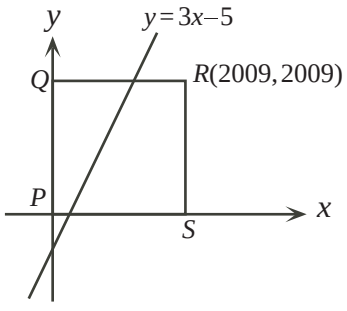
Part C: Each correct answer is worth 8.

21. A *lattice point* is a point with integer coordinates. (For example, $(1,4)$ is a lattice point but $(\frac{3}{2},4)$ is not.) The line $y = 3x - 5$ passes through square $PQRS$ as shown in the diagram. If the coordinates of R are $(2009, 2009)$, then the number of lattice points on the line which are inside the square is



(A) 666 **(B)** 667 **(C)** 668
(D) 669 **(E)** 670

22. Suppose that a, b and c are three numbers with

$$\begin{aligned} a+b &= 3 \\ ac+b &= 18 \\ bc+a &= 6 \end{aligned}$$

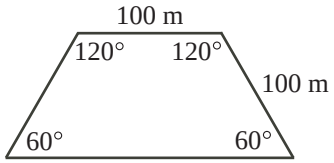
The value of c is

(A) 2 **(B)** 11 **(C)** 3 **(D)** 6 **(E)** 7

23. Angela and Barry share a piece of land. The ratio of the area of Angela's portion to the area of Barry's portion is 3 : 2. They each grow corn and peas on their piece of land. The entire piece of land is covered by corn and peas in the ratio 7 : 3. On Angela's portion of the land, the ratio of corn to peas is 4 : 1. What is the ratio of corn to peas for Barry's portion?

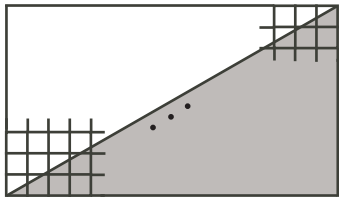
(A) 11 : 9 **(B)** 2 : 3 **(C)** 3 : 2 **(D)** 3 : 7 **(E)** 1 : 4

24. The field shown has been planted uniformly with wheat. At harvest, the wheat at any point in the field is brought to the nearest point on the field's perimeter. The fraction of the crop that is brought to the longest side is



(A) $\frac{1}{3}$ **(B)** $\frac{5}{12}$ **(C)** $\frac{1}{2}$
(D) $\frac{2}{5}$ **(E)** $\frac{4}{9}$

25. Unit squares are arranged to form a rectangular grid that is m units wide and n units tall, where m and n are positive integers with $2n < m < 3n$. The region below one of the diagonals of the rectangle is shaded as shown. For certain pairs m and n, there is a unit square in the grid that is not completely shaded but whose shaded area is greater than 0.999. The smallest possible value of mn for which this is true satisfies



(A) $496 \leq mn \leq 500$
(B) $501 \leq mn \leq 505$
(C) $506 \leq mn \leq 510$
(D) $511 \leq mn \leq 515$
(E) $516 \leq mn \leq 520$

Canadian Mathematics Competition

For students...

Thank you for writing the 2009 Cayley Contest!
In 2008, more than 83 000 students around the world registered to write the Pascal, Cayley and Fermat Contests.

Encourage your teacher to register you for the Galois Contest which will be written on April 8, 2009.
Visit our website

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- **More information about the Galois Contest**
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- **Find your school results**

Canadian Mathematics Competition

An activity of the Centre for Education in Mathematics and Computing, University of Waterloo, Waterloo, Ontario

Cayley Contest *(Grade 10)*

Tuesday, February 19, 2008

Time: 60 minutes

Calculators are permitted

Instructions

1. Do not open the Contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper left corner.
5. **Be certain that you code your name, age, sex, grade, and the Contest you are writing in the response form. Only those who do so can be counted as official contestants.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
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Scoring:	There is *no penalty* for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. What is the value of $3^2 - 2^2 + 1^2$?

   (A) 8 **(B)** -2 **(C)** 10 **(D)** -5 **(E)** 6

2. $\dfrac{\sqrt{25-16}}{\sqrt{25}-\sqrt{16}}$ is equal to

   (A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

3. Which of the following numbers is closest to 1?

   (A) $\frac{3}{4}$ **(B)** 1.2 **(C)** 0.81 **(D)** $1\frac{1}{3}$ **(E)** $\frac{7}{10}$

4. A bag contains 5 red, 6 green, 7 yellow, and 8 blue jelly beans. A jelly bean is selected at random. What is the probability that it is blue?

   (A) $\frac{5}{26}$ **(B)** $\frac{3}{13}$ **(C)** $\frac{7}{26}$ **(D)** $\frac{4}{13}$ **(E)** $\frac{6}{13}$

5. The 5-digit number 52 28$\square$ is a multiple of 6. Which digit is represented by $\square$?

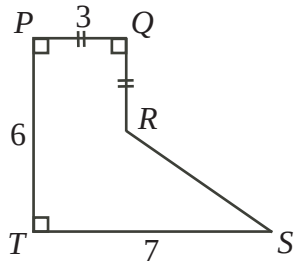
   (A) 0 **(B)** 3 **(C)** 4 **(D)** 6 **(E)** 8

6. If $\dfrac{40}{x} - 1 = 19$, then x is equal to

   (A) -1 **(B)** $\frac{1}{2}$ **(C)** 1 **(D)** 2 **(E)** -2

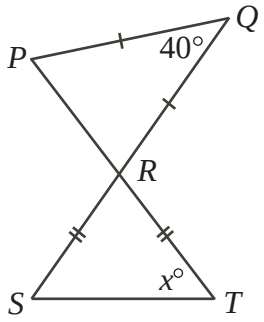
7. In the diagram, what is the perimeter of polygon $PQRST$?

   (A) 24 **(B)** 23 **(C)** 25 **(D)** 26 **(E)** 27



8. In the diagram, PRT and QRS are straight lines. What is the value of x?

   (A) 45 **(B)** 50 **(C)** 55 **(D)** 60 **(E)** 65

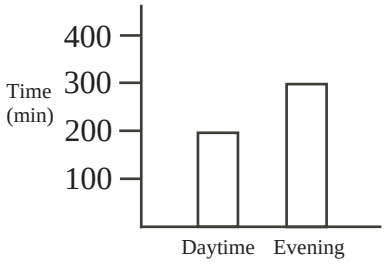


9. If $a = 7$ and $b = 13$, the number of even positive integers less than ab is

   (A) $\dfrac{ab-1}{2}$ **(B)** $\dfrac{ab}{2}$ **(C)** $ab - 1$ **(D)** $\dfrac{a+b}{4}$ **(E)** $(a-1)(b-1)$

10. Vivian's cell phone bill includes the graph showing her cell phone use for the month. She is charged

- \$20 per month, plus
- 10¢ per minute for daytime calls, plus
- 5¢ per minute for evening calls after the first 200 evening minutes. (The first 200 evening minutes are free.)



What is her total cell phone bill for the month shown?

(A) \$25 **(B)** \$40 **(C)** \$45 **(D)** \$70 **(E)** \$75

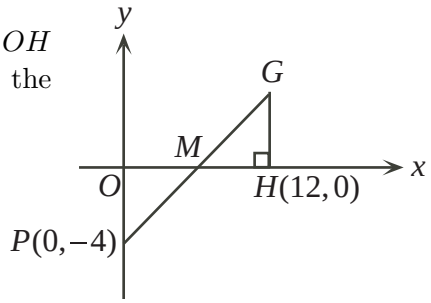
Part B: Each correct answer is worth 6.

11. Lex has \$2.65. He has only dimes (worth \$0.10 each) and quarters (worth \$0.25 each). If Lex has more quarters than dimes, how many coins does he have in total?

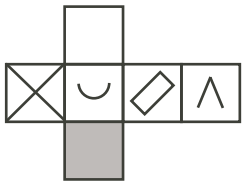
(A) 12 **(B)** 13 **(C)** 16 **(D)** 19 **(E)** 22

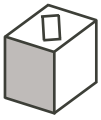
12. The line from G through the midpoint M of OH intersects the y-axis at $P(0,-4)$. What are the coordinates of G?

(A) $(12,3)$ **(B)** $(12,7)$ **(C)** $(12,5)$ **(D)** $(12,6)$ **(E)** $(12,4)$



13. The diagram shows a piece of cardboard that can be folded to make a cube. The cardboard has designs on one side only. Which one of the following cubes can be made from this cardboard?



(A)  **(B)**  **(C)** 

(D)  **(E)** 

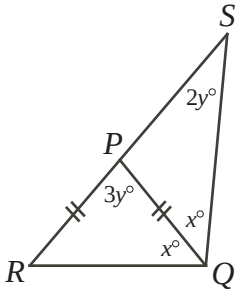
14. The first term of a sequence is 20.
If a term in the sequence is t and t is even, the next term is $\frac{1}{2}t$.
If a term in the sequence is t and t is odd, the next term is $3t+1$.
Therefore, the first three terms in the sequence are 20, 10, 5.
What is the 10th term of the sequence?

(A) 2 **(B)** 4 **(C)** 5 **(D)** 1 **(E)** 8

15. If x and y are two-digit positive integers with $xy = 555$, what is $x + y$?

(A) 52 **(B)** 116 **(C)** 66 **(D)** 555 **(E)** 45

16. In the diagram, P is on RS so that QP bisects $\angle SQR$. Also, $PQ = PR$, $\angle RSQ = 2y°$, and $\angle RPQ = 3y°$. The measure of $\angle RPQ$ is



(A) 90° **(B)** 108° **(C)** 120°
(D) 60° **(E)** 72°

17. If $3 \leq p \leq 10$ and $12 \leq q \leq 21$, then the difference between the largest and smallest possible values of $\frac{p}{q}$ is

(A) $\frac{29}{42}$ **(B)** $\frac{29}{5}$ **(C)** $\frac{19}{70}$ **(D)** $\frac{19}{12}$ **(E)** $\frac{19}{84}$

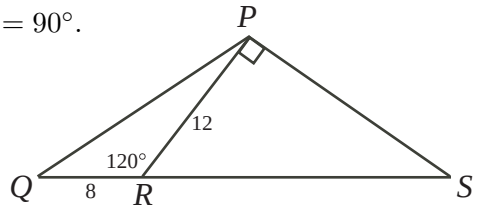
18. In the board game "Silly Bills", there are \$1, \$2 and \$3 bills.
There are 11 more \$2 bills than \$1 bills.
There are 18 fewer \$3 bills than \$1 bills.
If there is \$100 in total, then how many \$1 bills are there in the board game?

(A) 11 **(B)** 14 **(C)** 22 **(D)** 33 **(E)** 40

19. A box contains apple and pears.
An equal number of apples and pears are rotten.
$\frac{2}{3}$ of all of the apples are rotten.
$\frac{3}{4}$ of all of the the pears are rotten.
What fraction of the total number of pieces of fruit in the box is rotten?

(A) $\frac{17}{24}$ **(B)** $\frac{7}{12}$ **(C)** $\frac{5}{8}$ **(D)** $\frac{12}{17}$ **(E)** $\frac{5}{7}$

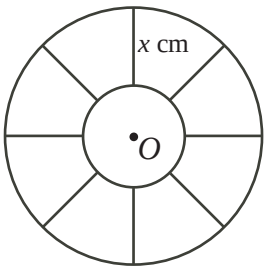
20. In the diagram, R is on QS and $QR = 8$.
Also, $PR = 12$, $\angle PRQ = 120°$, and $\angle RPS = 90°$.
What is the area of $\triangle QPS$?



(A) $72\sqrt{3}$ **(B)** 72 **(C)** 36
(D) $60\sqrt{3}$ **(E)** $96\sqrt{3}$

Part C: Each correct answer is worth 8.

21. The circular window shown in the diagram has nine panes of equal area. The inner circular pane has radius 20 cm and the same centre, O, as the outer circle. The eight lines separating the outer panes are of equal length, x cm, and all, if extended, would pass through O. What is the value of x, to the nearest tenth?



(A) 40.0 **(B)** 36.6 **(C)** 30.0
(D) 20.0 **(E)** 43.2

22. Suppose $N = 1 + 11 + 101 + 1001 + 10001 + \ldots + 1\overbrace{000\ldots00}^{50\text{ zeroes}}1$.
When N is calculated and written as a single integer, the sum of its digits is

(A) 50 **(B)** 99 **(C)** 55 **(D)** 58 **(E)** 103

23. If x and y are integers with $(y-1)^{x+y} = 4^3$, then the number of possible values for x is

(A) 8 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

24. A cube has edges of length 1 cm and has a dot marked in the centre of the top face. The cube is sitting on a flat table. The cube is rolled, without lifting or slipping, in one direction so that at least two of its vertices are always touching the table. The cube is rolled until the dot is again on the top face. The length, in centimetres, of the path travelled by the dot is

(A) π **(B)** 2π **(C)** $\sqrt{2}\pi$ **(D)** $\sqrt{5}\pi$ **(E)** $\left(\frac{1+\sqrt{5}}{2}\right)\pi$

25. The average value of $(a-b)^2 + (b-c)^2 + (c-d)^2 + (d-e)^2 + (e-f)^2 + (f-g)^2$ over all possible arrangements (a, b, c, d, e, f, g) of the seven numbers 1, 2, 3, 11, 12, 13, 14 is

(A) 398 **(B)** 400 **(C)** 396 **(D)** 392 **(E)** 394

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- **Find your school results**

Canadian Mathematics Competition

An activity of the Centre for Education in Mathematics and Computing, University of Waterloo, Waterloo, Ontario

Cayley Contest (Grade 10)

Tuesday, February 20, 2007

Time: 60 minutes

Calculators are permitted

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Scoring:	There is *no penalty* for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. The value of $8 + 2(3^2)$ is

   (A) 26 **(B)** 90 **(C)** 41 **(D)** 44 **(E)** 60

2. The value of $\dfrac{7+21}{14+42}$ is

   (A) $\frac{1}{3}$ **(B)** $\frac{1}{6}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{2}{3}$ **(E)** 1

3. If $3x - 2x + x = 3 - 2 + 1$, then x equals

   (A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

4. The table shows the pay Leona earned for two different shifts at the same fixed hourly rate. How much will she earn for a five hour shift at this rate?

Shift	Total Pay
3 hours	\$24.75
6 hours	\$49.50

   (A) \$43.75 **(B)** \$46.25 **(C)** \$38.75
   (D) \$36.25 **(E)** \$41.25

5. $\frac{1}{4}$ of 100 is equal to

   (A) 20% of 200 **(B)** 10% of 250 **(C)** 15% of 100 **(D)** 25% of 50 **(E)** 5% of 300

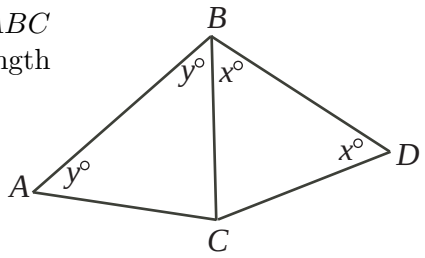
6. If $a = 2$ and $b = 5$, which of the following expressions has the greatest value?

   (A) $\dfrac{a}{b}$ **(B)** $\dfrac{b}{a}$ **(C)** $a - b$ **(D)** $b - a$ **(E)** $\frac{1}{2}a$

7. The mean (average) of 6, 9 and 18 is equal to the mean (average) of 12 and y. What is the value of y?

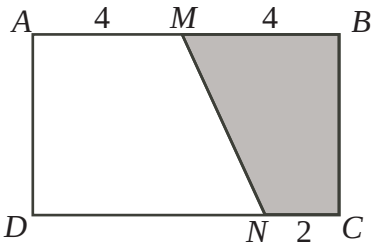
   (A) 22 **(B)** 21 **(C)** 10 **(D)** 11 **(E)** 5

8. In the diagram, triangles ABC and CBD are isosceles. The perimeter of $\triangle CBD$ is 19, the perimeter of $\triangle ABC$ is 20, and the length of BD is 7. What is the length of AB?



   (A) 5 **(B)** 6 **(C)** 7
   (D) 8 **(E)** 9

9. In the diagram, the area of rectangle $ABCD$ is 40. The area of $MBCN$ is



   (A) 15 **(B)** 10 **(C)** 30
   (D) 12 **(E)** 16

10. The first term in a sequence is x. Each of the following terms is obtained by doubling the previous term and then adding 4. If the third term is 52, then x equals

(A) 7 **(B)** 8 **(C)** 9 **(D)** 10 **(E)** 11

Part B: Each correct answer is worth 6.

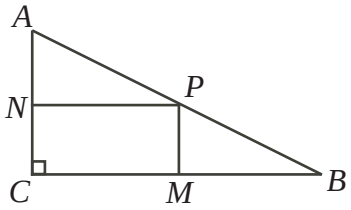
11. Ivan trained for a cross-country meet.
On Monday, he ran a certain distance.
On Tuesday, he ran twice as far as he ran on Monday.
On Wednesday, he ran half as far as he ran on Tuesday.
On Thursday, he ran half as far as he ran on Wednesday.
On Friday, he ran twice as far as he ran on Thursday.
If the shortest distance that he ran on any of the five days is 5 km, how far did he run in total?

(A) 55 km **(B)** 25 km **(C)** 27.5 km **(D)** 17.5 km **(E)** 50 km

12. The point $(0, 0)$ is reflected in the vertical line $x = 1$. When its image is then reflected in the line $y = 2$, the resulting point is

(A) $(0, 0)$ **(B)** $(2, 0)$ **(C)** $(4, 4)$ **(D)** $(2, 2)$ **(E)** $(2, 4)$

13. In the diagram, $\triangle ABC$ is right-angled at C. Also, points M, N and P are the midpoints of sides BC, AC and AB, respectively. If the area of $\triangle APN$ is 2 cm^2, then the area of $\triangle ABC$ is



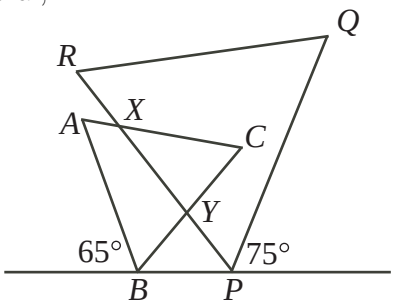
(A) 8 cm^2 **(B)** 16 cm^2 **(C)** 6 cm^2
(D) 4 cm^2 **(E)** 12 cm^2

14. If $\dfrac{3}{x-3} + \dfrac{5}{2x-6} = \dfrac{11}{2}$, then the value of $2x - 6$ is

(A) 2 **(B)** 12 **(C)** 6 **(D)** 8 **(E)** 10

15. In the diagram, if $\triangle ABC$ and $\triangle PQR$ are equilateral, then $\angle CXY$ equals

(A) 30° **(B)** 35° **(C)** 40°
(D) 45° **(E)** 50°



16. At Springfield University, there are 10 000 students, and there are as many male students as female students. Each student is enrolled either in the Arts program or Science program (but not in both); 60% of the students are in the Arts program. Also, 40% of the Science students are male. To the nearest percent, what percentage of the Arts students are female?

(A) 50% **(B)** 52% **(C)** 26% **(D)** 65% **(E)** 43%

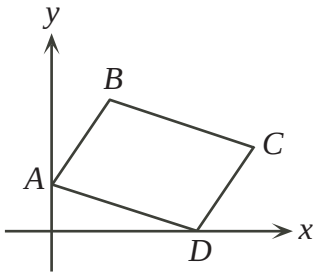
17. On an island there are two types of inhabitants: Heroes who always tell the truth and Villains who always lie. Four inhabitants are seated around a table. When each is asked "Are you a Hero or a Villain?", all four reply "Hero". When asked "Is the person on your right a Hero or a Villain?", all four reply "Villain". How many Heroes are present?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

18. There are a certain number of red balls, green balls and blue balls in a bag. Of the balls in the bag, $\frac{1}{3}$ are red and $\frac{2}{7}$ are blue. The number of green balls in the bag is 8 less than twice the number of blue balls. The number of green balls in the bag is

(A) 12 **(B)** 16 **(C)** 20 **(D)** 24 **(E)** 28

19. In the diagram, the four points have coordinates $A(0,1)$, $B(1,3)$, $C(5,2)$, and $D(4,0)$. What is the area of quadrilateral $ABCD$?



(A) 9 **(B)** 3 **(C)** 6
(D) $\sqrt{85}$ **(E)** $2\sqrt{5}+2\sqrt{17}$

20. What is the largest integer n for which $3(n^{2007}) < 3^{4015}$?

(A) 2 **(B)** 3 **(C)** 6 **(D)** 8 **(E)** 9

Part C: Each correct answer is worth 8.

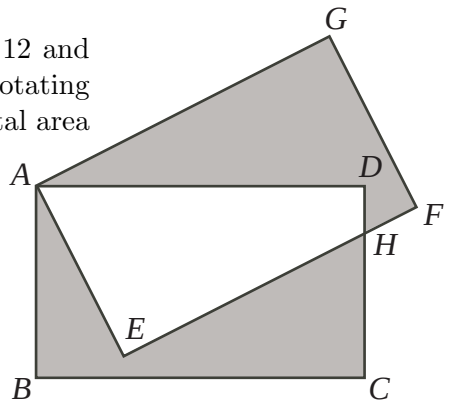
21. In a soccer league with 6 teams (P, Q, R, S, T, W), each team must eventually play each other team exactly once. So far, P has played one match, Q has played 2 matches, R has played 3 matches, S has played 4 matches, and T has played 5 matches. How many matches has W played so far?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

22. Five positive integers are listed in increasing order. The difference between any two consecutive numbers in the list is three. The fifth number is a multiple of the first number. How many different such lists of five integers are there?

(A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

23. In the diagram, $ABCD$ is rectangle with $AB = 12$ and $BC = 18$. Rectangle $AEFG$ is formed by rotating $ABCD$ about A through an angle of 30°. The total area of the shaded regions is closest to

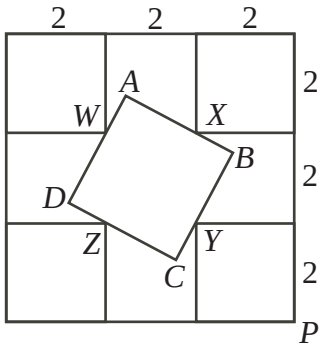


(A) 202.8 **(B)** 203.1 **(C)** 203.4
(D) 203.7 **(E)** 204.0

24. The number 8 is the sum and product of the numbers in the collection of four positive integers $\{1, 1, 2, 4\}$, since $1 + 1 + 2 + 4 = 8$ and $1 \times 1 \times 2 \times 4 = 8$. The number 2007 can be made up from a collection of n positive integers that multiply to 2007 and add to 2007. What is the smallest value of n with $n > 1$?

(A) 1171 **(B)** 1337 **(C)** 1551 **(D)** 1777 **(E)** 1781

25. In the diagram, four squares of side length 2 are placed in the corners of a square of side length 6. Each of the points W, X, Y, and Z is a vertex of one of the small squares. Square $ABCD$ can be constructed with sides passing through W, X, Y, and Z. The maximum possible distance from A to P is closest to



(A) 5.2 **(B)** 5.4 **(C)** 5.6

(D) 5.8 **(E)** 6.0

Canadian Mathematics Competition

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Thank you for writing the 2007 Cayley Contest!
In 2006, more than 90 000 students around the world registered to write the Pascal, Cayley and Fermat Contests.

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Canadian Mathematics Competition

An activity of the Centre for Education in Mathematics and Computing, University of Waterloo, Waterloo, Ontario

Cayley Contest *(Grade 10)*

Wednesday, February 22, 2006

Time: 60 minutes

Calculators are permitted

Instructions

1. Do not open the Contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper left corner.
5. **Be certain that you code your name, age, sex, grade, and the Contest you are writing in the response form. Only those who do so can be counted as official contestants.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor tells you to begin, you will have *sixty* minutes of working time.

Scoring:	There is *no penalty* for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. The value of $\frac{1}{2} + \left(\frac{1}{2} \times \frac{1}{2}\right)$ is

   (A) $\frac{3}{8}$ **(B)** 1 **(C)** $\frac{1}{6}$ **(D)** $\frac{1}{4}$ **(E)** $\frac{3}{4}$

2. The value of $\left(\sqrt{100} - \sqrt{36}\right)^2$ is

   (A) 16 **(B)** 256 **(C)** 8 **(D)** 1024 **(E)** 4096

3. The value of $43 - 41 + 39 - 37 + 35 - 33 + 31 - 29$ is

   (A) 8 **(B)** 6 **(C)** 10 **(D)** 12 **(E)** 16

4. If $a = -3$ and $b = 2$, the value of $a(b - 3)$ is

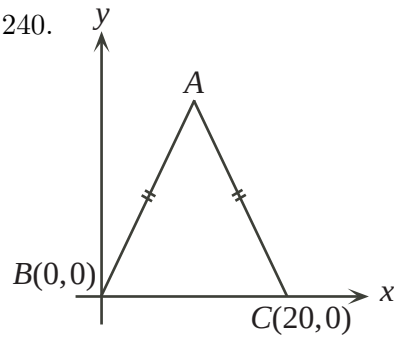
   (A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

5. In the four term sequence $0.001,\ 0.02,\ 0.4,\ x$, each term after the first is found by multiplying the previous term by the same number. What is the value of x?

   (A) 0.8 **(B)** 8 **(C)** 80 **(D)** 8.8 **(E)** 0.08

6. In the diagram, $\triangle ABC$ is isosceles and its area is 240. The y-coordinate of A is

   (A) 6 **(B)** 12 **(C)** 18 **(D)** 24 **(E)** 48

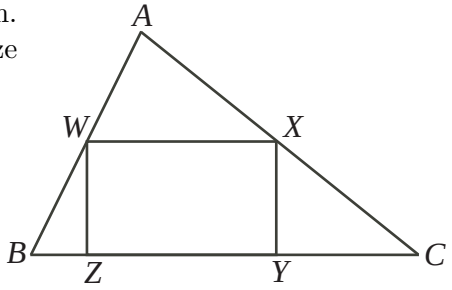


7. If $\dfrac{6}{x+1} = \dfrac{3}{2}$, then x equals

   (A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

8. A rectangle is drawn inside $\triangle ABC$, as shown. If $\angle BWZ = 22°$ and $\angle CXY = 65°$, then the size of $\angle BAC$ is

   (A) 87° **(B)** 90° **(C)** 93° **(D)** 104° **(E)** 82°



9. The lengths of the three sides of a triangle are 7, $x + 4$ and $2x + 1$. The perimeter of the triangle is 36. What is the length of the longest side of the triangle?

   (A) 7 **(B)** 12 **(C)** 17 **(D)** 15 **(E)** 16

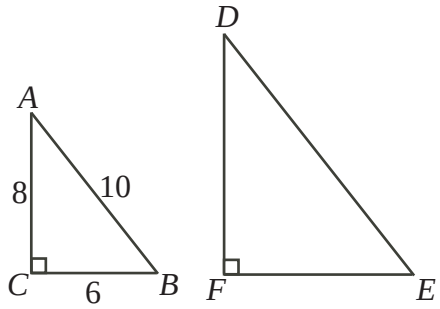
10. A class of 30 students recently wrote a test. If 20 students scored 80, 8 students scored 90, and 2 students scored 100, then the class average on this test was

(A) 90 **(B)** 84 **(C)** 82 **(D)** 86 **(E)** 88

Part B: Each correct answer is worth 6.

11. $\triangle ABC$ has side lengths 6, 8 and 10, as shown. Each of the side lengths of $\triangle ABC$ is increased by 50%, forming a new triangle, $\triangle DEF$. The area of $\triangle DEF$ is

(A) 24 **(B)** 48 **(C)** 108 **(D)** 12 **(E)** 54



12. From 7:45 p.m. to 9:30 p.m., Jim drove a distance of 84 km at a constant speed. What was this speed, in km/h?

(A) 60 **(B)** 80 **(C)** 112 **(D)** 63 **(E)** 48

13. If $x + 1 = y - 8$ and $x = 2y$, then the value of $x + y$ is

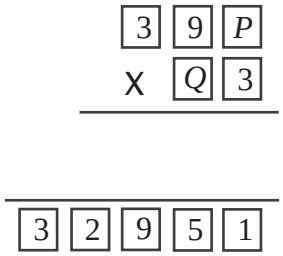
(A) -18 **(B)** 0 **(C)** -9 **(D)** -27 **(E)** -36

14. If $x = -3$, which of the following expressions has the smallest value?

(A) $x^2 - 3$ **(B)** $(x - 3)^2$ **(C)** x^2 **(D)** $(x + 3)^2$ **(E)** $x^2 + 3$

15. In the multiplication shown, P and Q each represent a single digit, and the product is 32 951. What is the value of $P + Q$?

(A) 14 **(B)** 12 **(C)** 15 **(D)** 13 **(E)** 11



16. In 2004, Gerry downloaded 200 songs. In 2005, Gerry downloaded 360 songs at a cost per song which was 32 cents less than in 2004. Gerry's *total* cost each year was the same. The cost of downloading the 360 songs in 2005 was

(A) \$144.00 **(B)** \$108.00 **(C)** \$80.00 **(D)** \$259.20 **(E)** \$72.00

17. If w is a positive integer and $w^3 = 9w$, then w^5 is equal to

(A) 59 049 **(B)** 243 **(C)** 1024 **(D)** 3125 **(E)** 32

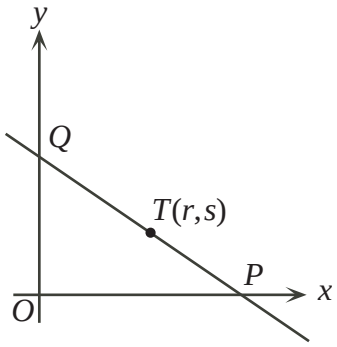
18. In a right-angled triangle, the sum of the squares of the three side lengths is 1800. The length of its hypotenuse is

(A) $\sqrt{1800}$ **(B)** $\frac{1}{2}\sqrt{1800}$ **(C)** 90 **(D)** 30 **(E)** 45

19. In a bin at the Cayley Convenience Store, there are 200 candies. Of these candies, 90% are black and the rest are gold. After Yehudi eats some of the black candies, 80% of the remaining candies in the bin are black. How many black candies did Yehudi eat?

(A) 2 **(B)** 20 **(C)** 40 **(D)** 100 **(E)** 160

20. The line $y = -\frac{3}{4}x + 9$ crosses the x-axis at P and the y-axis at Q. Point $T(r, s)$ is on line segment PQ. If the area of $\triangle POQ$ is three times the area of $\triangle TOP$, then the value of $r + s$ is



(A) 7 **(B)** 10 **(C)** 11 **(D)** 14 **(E)** 18

Part C: Each correct answer is worth 8.

21. If p, q and r are positive integers and $p + \dfrac{1}{q + \dfrac{1}{r}} = \dfrac{25}{19}$, then q equals

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

22. A positive integer is called *multiplicatively perfect* if it is equal to the product of its proper divisors. For example, 10 is multiplicatively perfect since its proper divisors are 1, 2 and 5, and it is true that $1 \times 2 \times 5 = 10$. How many multiplicatively perfect integers are there between 2 and 30?

(A) 9 **(B)** 5 **(C)** 8 **(D)** 6 **(E)** 4

23. Quincy and Celine have to move 16 small boxes and 10 large boxes. The chart indicates the time that each person takes to move each type of box. They start moving the boxes at 9:00 a.m. The earliest time at which they can be finished moving all of the boxes is

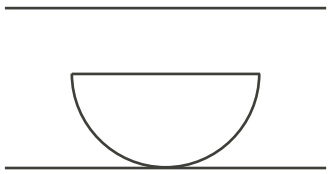
	Celine	Quincy
small box	2 min.	3 min.
large box	6 min.	5 min.

(A) 9:41 a.m. **(B)** 9:42 a.m. **(C)** 9:43 a.m. **(D)** 9:44 a.m. **(E)** 9:45 a.m.

24. Anne and Brenda play a game which begins with a pile of n toothpicks. They alternate turns with Anne going first. On each player's turn, she must remove 1, 3 or 4 toothpicks from the pile. The player who removes the last toothpick wins the game. For which of the following values of n does Brenda have a winning strategy? (In a game, a player has a winning strategy if, regardless of what the other player does, there are moves that she can make which *guarantee* that she will win.)

(A) 31 **(B)** 32 **(C)** 33 **(D)** 34 **(E)** 35

25. A semi-circle of radius 8 cm, rocks back and forth along a line. The distance between the line on which the semi-circle sits and the line above is 12 cm. As it rocks without slipping, the semi-circle touches the line above at two points. (When the semi-circle hits the line above, it immediately rocks back in the other direction.) The distance between these two points, in millimetres, is closest to



(A) 55 **(B)** 53 **(C)** 51

(D) 49 **(E)** 47

Canadian Mathematics Competition

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Thank you for writing the 2006 Cayley Contest!
In 2005, more than 90 000 students around the world registered to write the Pascal, Cayley and Fermat Contests.

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Canadian Mathematics Competition

An activity of the Centre for Education in Mathematics and Computing, University of Waterloo, Waterloo, Ontario

Cayley Contest (Grade 10)

Wednesday, February 23, 2005

Time: 60 minutes

Calculators are permitted.

Instructions

1. Do not open the Contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper left corner.
5. **Be certain that you code your name, age, sex, grade, and the Contest you are writing in the response form. Only those who do so can be counted as official contestants.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor tells you to begin, you will have *sixty* minutes of working time.

Scoring: There is *no penalty* for an incorrect answer.
Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. The expression $a+1+a-2+a+3+a-4$ is equal to

   (A) $10a$ **(B)** 0 **(C)** $4a-2$ **(D)** $4a+2$ **(E)** $-2a$

2. The value of $(\frac{4}{5})(\frac{5}{6})(\frac{6}{7})(\frac{7}{8})(\frac{8}{9})$ is

   (A) $\frac{4}{9}$ **(B)** 1 **(C)** $\frac{6}{7}$ **(D)** 36 **(E)** $\frac{36}{25}$

3. When 45 is divided by 7, the remainder is 3. What is the remainder when 70 is divided by 17?

   (A) 1 **(B)** 12 **(C)** 15 **(D)** 2 **(E)** 11

4. If $\dfrac{3}{x+10}=\dfrac{1}{2x}$, then x equals

   (A) $\frac{1}{2}$ **(B)** 10 **(C)** -4 **(D)** 2 **(E)** -8

5. A teacher writes five different possible values for $(5^2-4^2)^3$ on the board and asks her class to decide which is correct. The correct value is

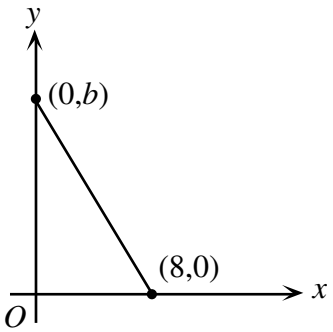
   (A) 1 **(B)** 8 **(C)** $11\,529$ **(D)** 216 **(E)** 729

6. Last week, a charity fundraiser had 8 volunteers who each worked 40 hours and who each raised \$18 per hour. This week, 12 volunteers, each working 32 hours, raised the same total amount of money. How much did each volunteer raise per hour this week?

   (A) $\$9$ **(B)** $\$12$ **(C)** $\$15$ **(D)** $\$21$ **(E)** $\$24$

7. In the diagram, the line segment has slope $-\frac{3}{2}$.
   The value of b is

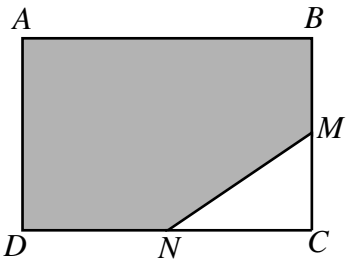
   (A) 10 **(B)** 12 **(C)** 6
   (D) 16 **(E)** 20



8. Jack went running last Saturday morning. He ran the first 12 km at 12 km/h and the second 12 km at 6 km/h. Jill ran the same route at a constant speed, and took the same length of time as Jack. Jill's speed in km/h was

   (A) 8 **(B)** 9 **(C)** 6 **(D)** 12 **(E)** 24

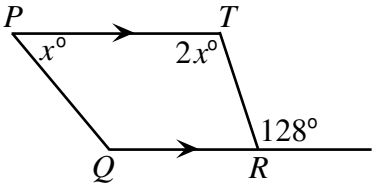
9. $ABCD$ is a rectangle, with M the midpoint of BC and N the midpoint of CD. If $CM = 4$ and $NC = 5$, what percent of the area of the rectangle is shaded?



(A) 70 **(B)** 78 **(C)** 80

(D) 87.5 **(E)** 75

10. In the diagram, PT is parallel to QR. What is the size of $\angle PQR$?



(A) 116° **(B)** 168° **(C)** 138°

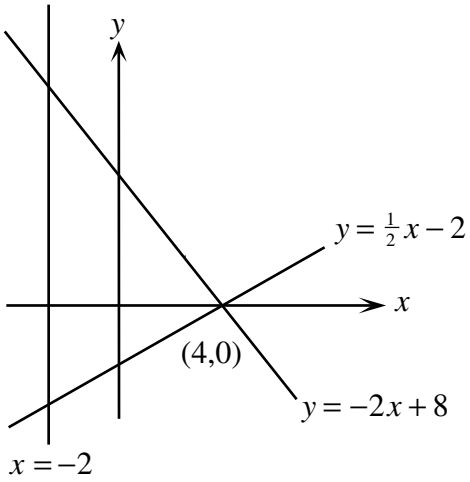
(D) 144° **(E)** 122°

Part B: Each correct answer is worth 6.

11. During a football game, Matt kicked the ball three times. His longest kick was 43 metres and the three kicks averaged 37 metres. If the other two kicks were the same length, the distance, in metres, that each travelled was

(A) 31 **(B)** 37 **(C)** 35 **(D)** 34 **(E)** 36

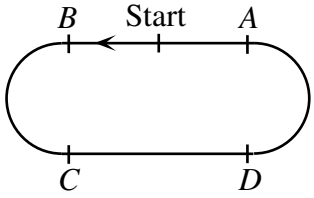
12. The lines $y = -2x + 8$ and $y = \frac{1}{2}x - 2$ meet at $(4, 0)$, as shown. The area of the triangle formed by these two lines and the line $x = -2$ is



(A) 15 **(B)** 27 **(C)** 30

(D) 36 **(E)** 45

13. A 400 m track is constructed so that the points A, B, C, and D divide the track into four segments of equal length. The Start is half-way between A and B. Andrew begins at the Start and walks at a steady rate of 1.4 m/s in a counter-clockwise direction. After exactly 30 minutes, to what point will Andrew be closest?



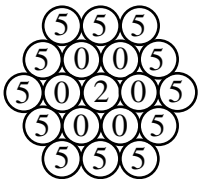
(A) A **(B)** B **(C)** C

(D) D **(E)** Start

14. If x is a *positive* integer less than 100, how many values of x make $\sqrt{1 + 2 + 3 + 4 + x}$ an integer?

(A) 6 **(B)** 7 **(C)** 8 **(D)** 9 **(E)** 10

15. Starting with the 2 in the centre, the number 2005 can be formed by moving from circle to circle only if the two circles are touching. How many different paths can be followed to form 2005?



(A) 36 **(B)** 24 **(C)** 12
(D) 18 **(E)** 6

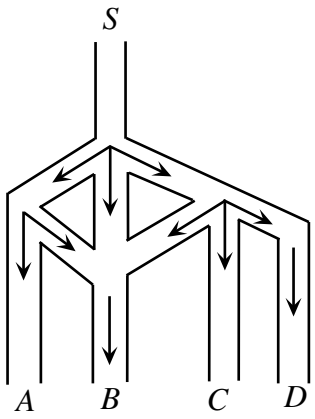
16. The *non-negative difference* between two numbers a and b is $a - b$ or $b - a$, whichever is greater than or equal to 0. For example, the non-negative difference between 24 and 64 is 40. In the sequence 88, 24, 64, 40, 24, ..., each number after the second is obtained by finding the non-negative difference between the previous 2 numbers. The sum of the first 100 numbers in this sequence is

(A) 496 **(B)** 760 **(C)** 752 **(D)** 776 **(E)** 405

17. 10^{100} is a googol. 1000^{100} equals

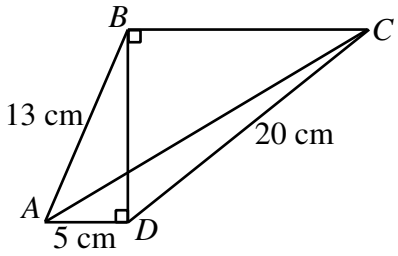
(A) 100 googol **(B)** 3 googol **(C)** googol$^{\text{googol}}$
(D) googol^2 **(E)** googol^3

18. Harry the Hamster is put in a maze, and he starts at point S. The paths are such that Harry can move forward only in the direction of the arrows. At any junction, he is equally likely to choose any of the forward paths. What is the probability that Harry ends up at B?



(A) $\frac{2}{3}$ **(B)** $\frac{13}{18}$ **(C)** $\frac{11}{18}$
(D) $\frac{1}{3}$ **(E)** $\frac{1}{4}$

19. In the diagram, $AB = 13$ cm, $DC = 20$ cm, and $AD = 5$ cm. The length of AC, to the nearest tenth of a centimetre, is



(A) 24.2 **(B)** 20.6 **(C)** 25.2
(D) 23.4 **(E)** 24.9

20. There are 81 cars in the CMC parking lot, which are all Acuras, Beetles, or Camrys. There are half as many Acuras as Beetles. The number of Camrys is 80% of the number of Acuras and Beetles together. How many of the 81 cars are Beetles?

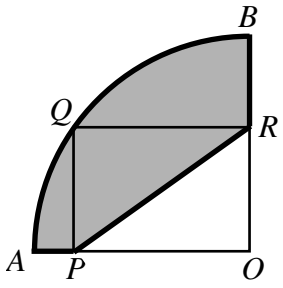
(A) 36 **(B)** 30 **(C)** 45 **(D)** 51 **(E)** 66

Part C: Each correct answer is worth 8.

21. In Yacleyland, the unit of money used is called the Yacley. There are only two denominations of paper money: the 17 Yacley bill and the 5 Yacley bill. How many different combinations of these bills total 453 Yacley?

(A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

22. In the diagram, AOB is a quarter circle of radius 10 and $PQRO$ is a rectangle of perimeter 26. The perimeter of the shaded region is



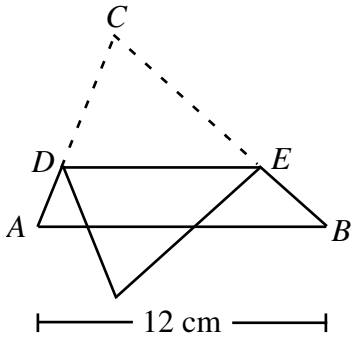
(A) $7 + 5\pi$ **(B)** $13 + 5\pi$ **(C)** $17 + 5\pi$

(D) $7 + 25\pi$ **(E)** $17 + 25\pi$

23. At 12:00 noon, Anna and Bill left home and walked in the same direction. Anna walked at 4 km/h and Bill walked at 3 km/h. At 12:15 their dog Dexter, running at 6 km/h, left home to run after them. The dog ran until it caught up to Anna, then it ran back to Bill. (In his excitement, Dexter lost no time in turning around once he reached Anna.) At what time did Bill meet Dexter on Dexter's way back?

(A) 1:00 p.m. **(B)** 1:15 p.m. **(C)** 12:45 p.m. **(D)** 1:05 p.m. **(E)** 12:50 p.m.

24. The base of a triangular piece of paper ABC is 12 cm long. The paper is folded down over the base, with the crease DE parallel to the base of the paper. The area of the triangle that projects below the base is 16% that of the area of the triangle ABC. The length of DE, in cm, is



(A) 9.6 **(B)** 8.4 **(C)** 7.2

(D) 4.8 **(E)** 6.96

25. The positive integers a, b and c satisfy $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c^2}$.

The sum of all possible values of $a \leq 100$ is

(A) 315 **(B)** 615 **(C)** 680 **(D)** 555 **(E)** 620

Canadian Mathematics Competition

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Thank you for writing the 2005 Cayley Contest!
In 2004, more than 83 000 students around the world registered to write the Pascal, Cayley and Fermat Contests.

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Canadian Mathematics Competition

An activity of The Centre for Education in Mathematics and Computing, University of Waterloo, Waterloo, Ontario

Cayley Contest *(Grade 10)*

Wednesday, February 18, 2004

Time: 1 hour

Calculators are permitted.

Instructions

1. Do not open the contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper right corner.
5. **Be certain that you code your name, age, sex, grade, and the contest you are writing on the response form. Only those who do so can be counted as official contestants.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. When you have decided on your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor instructs you to begin, you will have *sixty* minutes of working time.

Scoring:	There is *no penalty* for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. The value of $2^2+1^2+0^2+(-1)^2+(-2)^2$ is

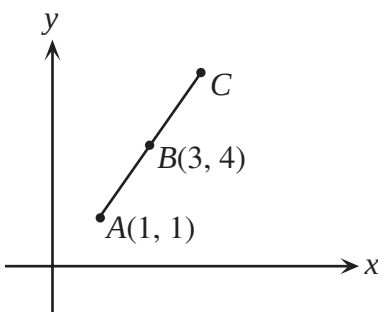
(**A**) 5 (**B**) -6 (**C**) 3 (**D**) 6 (**E**) 10

2. 25% of 2004 is equal to

(**A**) 50% of 4008 (**B**) 50% of 1002 (**C**) 100% of 1002
(**D**) 10% of 8016 (**E**) 20% of 3006

3. Point $B(3,4)$ is the midpoint of the line segment joining the points $A(1,1)$ and C. The coordinates of C are

(**A**) $(2,3)$ (**B**) $(2,2)$ (**C**) $(4,6)$
(**D**) $(5,8)$ (**E**) $(5,7)$



4. If $x+1-2+3-4=5-6+7-8$, the value of x is

(**A**) -2 (**B**) -1 (**C**) 0 (**D**) 1 (**E**) 2

5. In the sequence, each figure is made up of small squares of side length 1. What is the outer perimeter of the fifth figure in the sequence?

(**A**) 9 (**B**) 18 (**C**) 20 (**D**) 24 (**E**) 36

6. If $x+6y=17$, the value of $7x+42y$ is

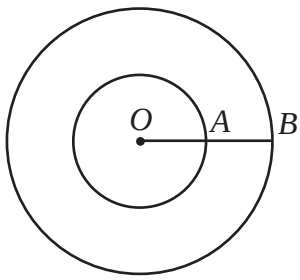
(**A**) 24 (**B**) 42 (**C**) 49 (**D**) 102 (**E**) 119

7. If $3^2+3^2+3^2=3^a$, the value of a is

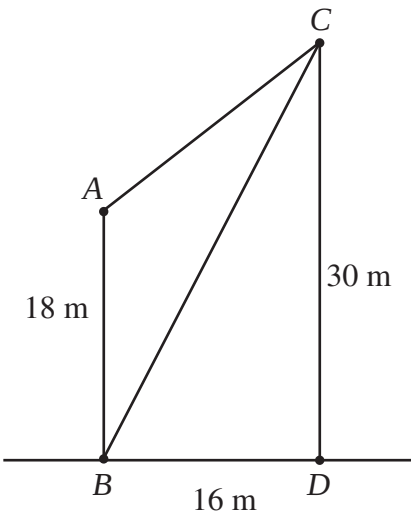
(**A**) 2 (**B**) 3 (**C**) 4 (**D**) 6 (**E**) 8

8. In the diagram, O is the centre of each circle. The circumferences of the circles are 24π and 14π. B is a point on the outer circle and OB intersects the inner circle at A. The length of AB is

(**A**) $\sqrt{10}$ (**B**) 5 (**C**) 7
(**D**) 10π (**E**) 3

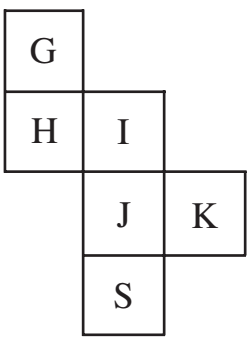


9. Two vertical towers, AB and CD, are located 16 m apart on flat ground, as shown. Tower AB is 18 m tall and tower CD is 30 m tall. Ropes are tied from A to C and from B to C. Assuming the ropes are taut, the total length of rope, in m, is



(**A**) 54 (**B**) 64 (**C**) 44
(**D**) 48 (**E**) 59

10. If the figure shown is folded to make a cube, what letter is opposite G?



(**A**) S (**B**) H (**C**) I
(**D**) J (**E**) K

Part B: Each correct answer is worth 6.

11. In the sequence of five numbers x, _____, 3, _____, 18, each number after the second is obtained by multiplying the two previous numbers. The value of x is

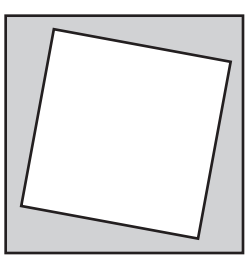
(**A**) $\frac{2}{3}$ (**B**) $\frac{3}{2}$ (**C**) 1 (**D**) -9 (**E**) -1

12. In the magic square, the sum of the three numbers in any row, column or diagonal is the same. The sum of the three numbers in any row is

$2x$	3	2
		-3
0	x	

(**A**) 0 (**B**) 1 (**C**) 3
(**D**) 7 (**E**) 9

13. In the diagram, a smaller square lies inside a larger square. The perimeter of the smaller square is 72 cm. The shaded area is 160 cm^2. The perimeter of the larger square, in cm, is

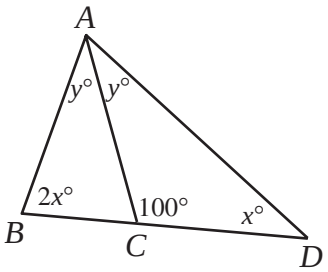


(**A**) 58 (**B**) 88 (**C**) 116
(**D**) 121 (**E**) 112

14. If x and y are positive numbers and the average of 4, 20 and x is equal to the average of y and 16, then the ratio $x:y$ is

(**A**) 3:2 (**B**) 2:3 (**C**) 1:1 (**D**) 2:5 (**E**) 5:2

15. In the diagram, B, C and D lie on a straight line, with $\angle ACD = 100^\circ$, $\angle ADB = x^\circ$, $\angle ABD = 2x^\circ$, and $\angle DAC = \angle BAC = y^\circ$. The value of x is



(**A**) 10 (**B**) 45 (**C**) 30
(**D**) 50 (**E**) 20

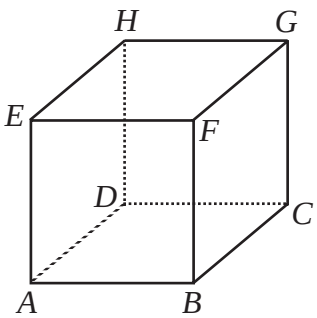
16. In a dice game, a player rolls two dice. His score is the larger of the two numbers on the dice. For example, if he rolls 3 and 5, his score is 5, and if he rolls 4 and 4, his score is 4. What is the probability that his score is 3 or less?

(**A**) $\frac{1}{4}$ (**B**) $\frac{7}{36}$ (**C**) $\frac{5}{36}$ (**D**) $\frac{1}{3}$ (**E**) $\frac{2}{9}$

17. The two whole numbers m and n satisfy $m+n=20$ and $\frac{1}{m}+\frac{1}{n}=\frac{5}{24}$. The product mn is equal to

(**A**) 72 (**B**) 36 (**C**) 48 (**D**) 96 (**E**) 24

18. In the diagram, $ABCDEFGH$ is a cube with an edge length of 12 cm. An ant sits on the cube at vertex A. The ant can only walk along the edges of the cube, and cannot walk along any edge more than once. What is the greatest distance that the ant can walk before it cannot continue?

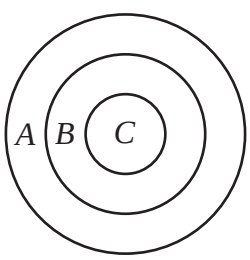


(**A**) 96 cm (**B**) 144 cm (**C**) 84 cm
(**D**) 108 cm (**E**) 132 cm

19. $\frac{1}{2}+\frac{2^1}{2^2}+\frac{2^2}{2^3}+\cdots+\frac{2^{2002}}{2^{2003}}+\frac{2^{2003}}{2^{2004}}$ is equal to

(**A**) 1002 (**B**) 501 (**C**) $\frac{1}{2^{2004}}$ (**D**) 2004 (**E**) $\frac{2004}{2^{2004}}$

20. An archery target has 3 regions, each worth a different value if it is hit. Three archers shoot two arrows each and record scores as follows:



First archer: 1 arrow in C and 1 arrow in A for a score of 15 points
Second archer: 1 arrow in C and 1 arrow in B for a score of 18 points
Third archer: 1 arrow in B and 1 arrow in A for a score of 13 points

If a fourth archer shoots 2 arrows into ring B, her score is

(**A**) 10 (**B**) 14 (**C**) 16 (**D**) 18 (**E**) 20

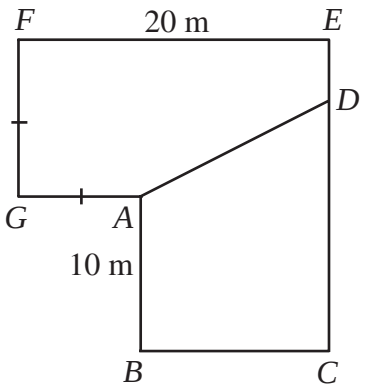
continued ...

Part C: Each correct answer is worth 8.

21. In a pack of construction paper, the numbers of blue and red sheets are originally in the ratio 2:7. Each day, Laura uses 1 blue sheet and 3 red sheets. One day, she uses 3 red sheets and the last blue sheet, leaving her with 15 red sheets. How many sheets of construction paper were in the pack originally?

(**A**) 144 (**B**) 252 (**C**) 135 (**D**) 270 (**E**) 105

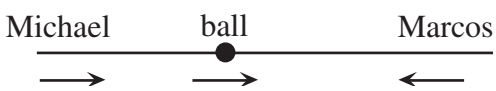
22. In the diagram, $ABCDEFG$ is a room having square corners, with $EF = 20$ m, $AB = 10$ m, and $AG = GF$. The total area of the room is $280\ \text{m}^2$. A wall is built from A to D creating two rooms of equal area. What is the distance, in metres, from C to D?



(**A**) 15 (**B**) $\frac{50}{3}$ (**C**) 12

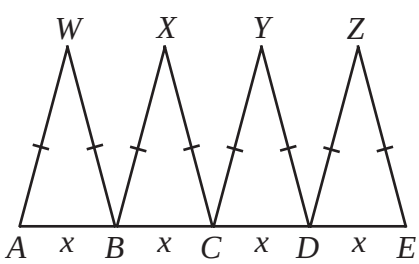
(**D**) 13 (**E**) $\frac{40}{3}$

23. A soccer ball rolls at 4 m/s towards Marcos in a direct line from Michael. The ball is 15 m ahead of Michael who is chasing it at 9 m/s. Marcos is 30 m away from the ball and is running towards it at 8 m/s. The distance between Michael and Marcos when the ball is touched for the first time by one of them is closest to



(**A**) 2.00 m (**B**) 2.25 m (**C**) 2.50 m
(**D**) 2.75 m (**E**) 3.00 m

24. Four identical isosceles triangles AWB, BXC, CYD, and DZE are arranged, as shown, with points A, B, C, D, and E lying on the same straight line. A new triangle is formed with sides the same lengths as AX, AY and AZ. If $AZ = AE$, the largest integer value of x such that the area of this new triangle is less than 2004 is



(**A**) 18 (**B**) 19 (**C**) 20
(**D**) 21 (**E**) 22

25. The number of positive integers x with $x \le 60$ such that each of the rational expressions

$$\frac{7x+1}{2}, \frac{7x+2}{3}, \frac{7x+3}{4}, \ldots, \frac{7x+300}{301}$$

is in lowest terms (i.e. in each expression, the numerator and denominator have no common factors) is

(**A**) 1 (**B**) 2 (**C**) 3 (**D**) 4 (**E**) 5

PUBLICATIONS

Students and parents who enjoy solving problems for fun and recreation may find the following publications of interest. They are an excellent resource for enrichment, problem solving and contest preparation.

Copies of Previous Canadian Mathematics Competitions

Copies of previous contests and solutions are available at no cost in both English and French at http://www.cemc.uwaterloo.ca

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- 10 topics
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- over 325 problems and full solutions
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Volume 3
- over 235 problems and full solutions
- 12 topics
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- 11 topics
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Faculty of Mathematics, Room 5181
University of Waterloo
Waterloo, ON N2L 3G1

Include your name, address (with postal code), and telephone number.

Cheques or money orders in Canadian funds should be made payable to "Centre for Education in Mathematics and Computing". In Canada, add $3.00 for the first item ordered for shipping and handling, plus $1.00 for each subsequent item. No Provincial Sales Tax is required, but 7% GST must be added. Orders ***outside of Canada ONLY***, add $10.00 for the first item ordered for shipping and handling, plus $2.00 for each subsequent item. **Prices for these publications will remain in effect until September 1, 2004.**

Canadian Mathematics Competition

An activity of The Centre for Education in Mathematics and Computing, University of Waterloo, Waterloo, Ontario

Cayley Contest (Grade 10)

Wednesday, February 19, 2003

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C.M.C. Contributors:

Manulife Financial

Time: 1 hour

Calculators are permitted.

Instructions

1. Do not open the contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
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8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor instructs you to begin, you will have *sixty* minutes of working time.

Scoring:	There is *no penalty* for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. The value of $\dfrac{3-(-3)}{2-1}$ is

   (A) 2 **(B)** 0 **(C)** 3 **(D)** 6 **(E)** -3

2. $17^2 - 15^2$ equals

   (A) 8^2 **(B)** 2^2 **(C)** 4^2 **(D)** 7^2 **(E)** 6^2

3. The integer 42 is

   (A) an odd number **(B)** a prime number **(C)** a perfect cube
   (D) divisible by 7 **(E)** a perfect square

4. If 5% of a number is 8, what is 25% of the same number?

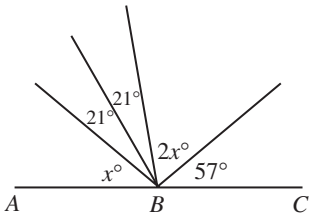
   (A) 40 **(B)** 0.1 **(C)** 320 **(D)** 10 **(E)** 200

5. The integer closest to the value of $\dfrac{3}{2} \times \dfrac{4}{9} + \dfrac{7}{2}$ is

   (A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

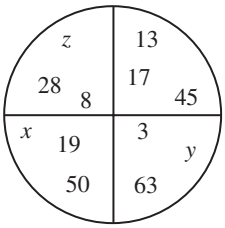
6. In the diagram, ABC is a straight line. The value of x is

   (A) 27 **(B)** 33 **(C)** 24
   (D) 87 **(E)** 81



7. In the diagram, the sum of the numbers in each quarter circle is the same. The value of $x + y + z$ is

   (A) 75 **(B)** 64 **(C)** 54
   (D) 171 **(E)** 300



8. An equilateral triangle has a side length of 20. If a square has the same perimeter as this triangle, the area of the square is

   (A) 25 **(B)** 400 **(C)** 225 **(D)** 60 **(E)** 100

9. If $\dfrac{1}{x+\frac{1}{5}}=\dfrac{5}{3}$, then x equals

(A) $\frac{2}{5}$ **(B)** $\frac{4}{5}$ **(C)** $\frac{1}{5}$ **(D)** $-\frac{2}{5}$ **(E)** $-\frac{22}{5}$

10. There are 2 girls and 6 boys playing a game. How many additional girls must join the game so that $\frac{5}{8}$ of the players are girls?

(A) 6 **(B)** 3 **(C)** 5 **(D)** 8 **(E)** 7

Part B: Each correct answer is worth 6.

11. Let $N=10^3+10^4+10^5+10^6+10^7+10^8+10^9$. The sum of the digits of N is

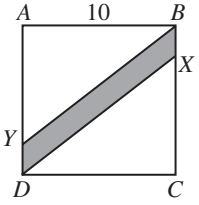
(A) 12 **(B)** 1 **(C)** 6 **(D)** 9 **(E)** 7

12. The points $A(a,1)$, $B(9,0)$ and $C(-3,4)$ lie on a straight line. The value of a is

(A) 3 **(B)** $\frac{8}{3}$ **(C)** $\frac{7}{2}$ **(D)** 6 **(E)** $\frac{5}{2}$

13. In the diagram, $ABCD$ is a square with a side length of 10. If $AY = CX = 8$, the area of the shaded region is

(A) 16 **(B)** 20 **(C)** 40
(D) 48 **(E)** 24

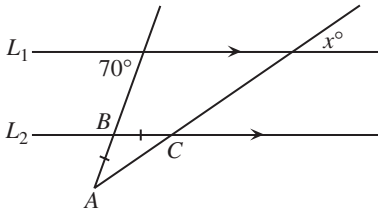


14. Carly takes three steps to walk the same distance as Jim walks in four steps. Each of Carly's steps covers 0.5 metres. How many metres does Jim travel in 24 steps?

(A) 16 **(B)** 9 **(C)** 36 **(D)** 12 **(E)** 18

15. In the diagram, line L_1 is parallel to line L_2 and $BA = BC$. The value of x is

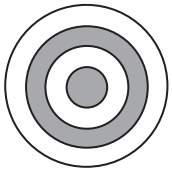
(A) 35 **(B)** 30 **(C)** 37.5
(D) 45 **(E)** 40



16. The value of $\dfrac{\left(4^{2003}\right)\left(3^{2002}\right)}{\left(6^{2002}\right)\left(2^{2003}\right)}$ is

(A) 1 **(B)** 2 **(C)** 12 **(D)** 4 **(E)** $\frac{1}{2}$

17. In the diagram, the four circles have a common centre, and have radii of 1, 2, 3, and 4. The ratio of the area of the shaded regions to the area of the largest circle is



(**A**) 5 : 8 (**B**) 1 : 4 (**C**) 7 : 16
(**D**) 1 : 2 (**E**) 3 : 8

18. If $496 = 2^m - 2^n$, where m and n are integers, then $m + n$ is equal to

(**A**) 13 (**B**) 9 (**C**) 4 (**D**) 14 (**E**) 5

19. The product of the digits of a four-digit number is 810. If none of the digits is repeated, the sum of the digits is

(**A**) 18 (**B**) 19 (**C**) 23 (**D**) 25 (**E**) 22

20. A car uses 8.4 litres of gas for every 100 km it is driven. A mechanic is able to modify the car's engine at a cost of \$400 so that it will only use 6.3 litres of gas per 100 km. The owner determines the minimum distance that she would have to drive to recover the cost of the modifications. If gas costs \$0.80 per litre, this distance, in kilometres, is between

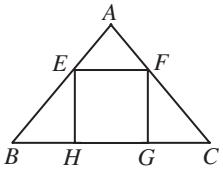
(**A**) 10 000 and 14 000 (**B**) 14 000 and 18 000 (**C**) 18 000 and 22 000
(**D**) 22 000 and 26 000 (**E**) 26 000 and 30 000

Part C: Each correct answer is worth 8.

21. Troye and Daniella are running at constant speeds in opposite directions around a circular track. Troye completes one lap every 56 seconds and meets Daniella every 24 seconds. How many seconds does it take Daniella to complete one lap?

(**A**) 32 (**B**) 36 (**C**) 40 (**D**) 48 (**E**) 42

22. In the diagram, ΔABC is isosceles with $AB = AC$ and $BC = 30$ cm. Square $EFGH$, which has a side length of 12 cm, is inscribed in ΔABC, as shown. The area of ΔAEF, in cm^2, is



(**A**) 27 (**B**) 54 (**C**) 51
(**D**) 48 (**E**) 60

23. A pyramid has a square base which has an area of 1440 cm^2. Each of the pyramid's triangular faces is identical and each has an area of 840 cm^2. The height of the pyramid, in cm, is

(**A**) $30\sqrt{2}$ (**B**) 40 (**C**) $20\sqrt{6}$ (**D**) $20\sqrt{3}$ (**E**) 30

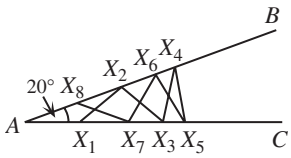
24. In how many ways can a, b, c, and d be chosen from the set $\{0, 1, 2, ..., 9\}$ so that $a < b < c < d$ and $a + b + c + d$ is a multiple of three?

(**A**) 54 (**B**) 64 (**C**) 63 (**D**) 90 (**E**) 72

continued ...

25. $\angle BAC$ is said to be "laceable" if distinct points $X_1, X_2, \ldots, X_{2n}$ can be found so that

- X_{2k-1} is on AC for each value of k,
- X_{2k} is on AB for each value of k, and
- $AX_1 = X_1X_2 = X_2X_3 = \cdots = X_{2n-1}X_{2n} = X_{2n}A$.



For example, the angle 20° is laceable, as shown. The number of laceable acute angles, whose sizes in degrees are integers, is

(**A**) 3 (**B**) 4 (**C**) 5
(**D**) 6 (**E**) 7

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PUBLICATIONS

Students and parents who enjoy solving problems for fun and recreation may find the following publications of interest. They are an excellent resource for enrichment, problem solving and contest preparation.

Copies of Previous Canadian Mathematics Competitions

Copies of previous contests and solutions are available at no cost in both English and French at http://www.cemc.uwaterloo.ca

Problems Problems Problems Books

Each volume is a collection of problems (multiple choice and full solution), grouped into 9 or more topics. Questions are selected from previous Canadian Mathematics Competition contests, and full solutions are provided for all questions. The price is $15. **(Available in English only.)**

Volume 1
- over 300 problems and full solutions
- 10 topics
- for students in Grades 9, 10, & 11
- French version of Volume 1 is available

Volume 2
- over 325 problems and full solutions
- 10 topics (different from Volume 1)
- for students in Grades 9, 10, & 11

Volume 3
- over 235 problems and full solutions
- 12 topics
- for senior high school students

Volume 4
- over 325 problems and full solutions
- 12 topics
- for students in Grades 7, 8, & 9

Volume 5
- over 200 problems and full solutions
- 9 topics (different from Volume 3)
- for senior high school students

Volume 6
- over 300 problems and full solutions
- 11 topics
- for students in Grades 7, 8, & 9

Volume 7


- over 300 problems and full solutions
- 12 topics
- for students in Grades 9 and 10

Volume 8


- over 200 problems and full solutions
- 10 topics
- for students in Grades 11 and 12

Problems and How To Solve Them - Volume 1

This book continues the collection of problems available for enrichment of students in grades 9, 10, and 11. Included for each of the eight chapters is a discussion on solving problems, with suggested approaches. There are more than 225 new problems, almost all from Canadian Mathematics Competitions, with complete solutions. The price is $20. **(Available in English only.)**

Orders should be addressed to: Canadian Mathematics Competition
Faculty of Mathematics, Room 5181
University of Waterloo
Waterloo, ON N2L 3G1

Include your name, address (with postal code), and telephone number.

Cheques or money orders in Canadian funds should be made payable to "Centre for Education in Mathematics and Computing". In Canada, add $3.00 for the first item ordered for shipping and handling, plus $1.00 for each subsequent item. No Provincial Sales Tax is required, but 7% GST must be added. Orders ***outside of Canada ONLY***, add $10.00 for the first item ordered for shipping and handling, plus $2.00 for each subsequent item. **Prices for these publications will remain in effect until September 1, 2003.**

Canadian Mathematics Competition

An activity of The Centre for Education in Mathematics and Computing, University of Waterloo, Waterloo, Ontario

Cayley Contest (Grade 10)

Wednesday, February 20, 2002

Time: 1 hour

Calculators are permitted, providing they are non-programmable and without graphic displays.

Instructions

1. Do not open the contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper right corner.
5. **Be certain that you code your name, age, sex, grade, and the contest you are writing on the response form. Only those who do so can be counted as official contestants.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. When you have decided on your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor instructs you to begin, you will have *sixty* minutes of working time.

Scoring:	There is *no penalty* for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. $5x+2(4+x)$ is equal to

   (**A**) $5x+8$ (**B**) $3x+6$ (**C**) $7x+8$ (**D**) $7x+6$ (**E**) $6x+8$

2. The value of $(2+3)^2-(2^2+3^2)$ is

   (**A**) 12 (**B**) 0 (**C**) 30 (**D**) 16 (**E**) -3

3. If $x=-3$, the numerical value of $x^2-4(x-5)$ is

   (**A**) 40 (**B**) 38 (**C**) -23 (**D**) 41 (**E**) -26

4. If n is $\frac{5}{6}$ of 240, then $\frac{2}{5}$ of n is

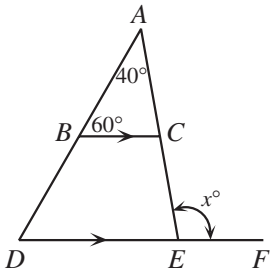
   (**A**) 288 (**B**) 80 (**C**) 96 (**D**) 200 (**E**) 500

5. The numerical value of $2^{-2}\times 2^{-1}\times 2^0\times 2^1\times 2^2$ is

   (**A**) 4 (**B**) 1 (**C**) 0 (**D**) $\frac{1}{4}$ (**E**) $\frac{1}{2}$

6. In the diagram, the value of x is

   (**A**) 130 (**B**) 120 (**C**) 110
   (**D**) 100 (**E**) 80



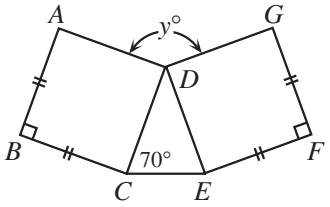
7. If the point $(-2, 4)$ is on a line with slope $\frac{1}{2}$, then the y-intercept of this line is

   (**A**) 5 (**B**) -4 (**C**) 3 (**D**) 0 (**E**) 8

8. After having played three basketball games, Megan had scored an average of 18 points per game. After her fourth game, her scoring average had dropped to 17 points per game. How many points did Megan score in her fourth game?

   (**A**) 18 (**B**) 17 (**C**) 16 (**D**) 15 (**E**) 14

9. In the diagram, $ABCD$ and $DEFG$ are squares with equal side lengths, and $\angle DCE = 70°$. The value of y is



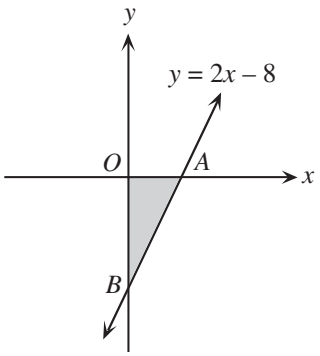
(**A**) 120 (**B**) 160 (**C**) 130
(**D**) 110 (**E**) 140

10. Faruq subtracted 5 from a number and then divided by 4. Next, he subtracted 4 from the original number and then divided by 5. He got the same final answer both times. The original number was

(**A**) 4 (**B**) 15 (**C**) 9 (**D**) 20 (**E**) -9

Part B: Each correct answer is worth 6.

11. In the diagram, the line with equation $y = 2x - 8$ crosses the x-axis at A and the y-axis at B. The area of ΔAOB is

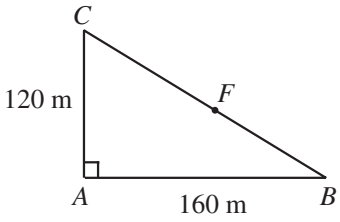


(**A**) 8 (**B**) 16 (**C**) 12
(**D**) 32 (**E**) 4

12. A compact disc originally sells for \$10.00. If the price of the compact disc is increased by 40% and this new price is later decreased by 30%, what is the final price?

(**A**) \$9.80 (**B**) \$17.00 (**C**) \$9.00 (**D**) \$19.80 (**E**) \$9.60

13. In the diagram, ABC represents a triangular jogging path. Jack jogs along the path from A to B to F. Jill jogs from A to C to F. Each jogs the same distance. The distance from F to B, in metres, is

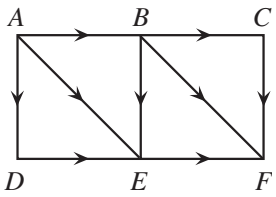


(**A**) 40 (**B**) 120 (**C**) 100
(**D**) 80 (**E**) 200

14. If $a(c+d)+b(c+d)=42$ and $c+d=3$, what is the value of $a+b+c+d$?

(**A**) 14 (**B**) 56 (**C**) 3 (**D**) 17 (**E**) 39

15. In the grid shown, it is only possible to travel along an edge in the direction indicated by the arrow. The number of different paths from A to F is



(**A**) 9 (**B**) 5 (**C**) 3
(**D**) 6 (**E**) 4

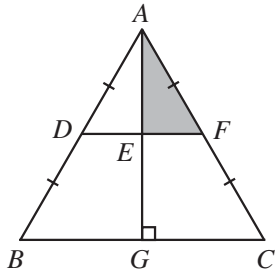
16. If the product of four consecutive positive integers is 358 800, then the sum of these four integers is

(**A**) 102 (**B**) 98 (**C**) 94 (**D**) 90 (**E**) 106

17. A "double-single" number is a three-digit number made up of two identical digits followed by a different digit. For example, 553 is a double-single number. How many double-single numbers are there between 100 and 1000?

(**A**) 81 (**B**) 18 (**C**) 72 (**D**) 64 (**E**) 90

18. In the diagram, triangle ABC is isosceles with $AB = AC$, and AG is perpendicular to BC. Point D is the midpoint of AB, point F is the midpoint of AC, and E is the point of intersection of DF and AG. What fraction of the area of ΔABC does the shaded area represent?



(**A**) $\frac{1}{12}$ (**B**) $\frac{1}{6}$ (**C**) $\frac{1}{4}$

(**D**) $\frac{1}{10}$ (**E**) $\frac{1}{8}$

19. The sum of the digits of the integer equal to $777\,777\,777\,777\,777^2 - 222\,222\,222\,222\,223^2$ is

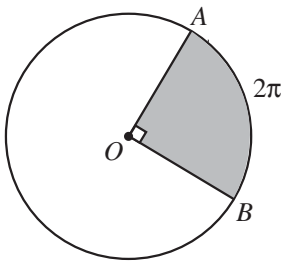
(**A**) 148 (**B**) 84 (**C**) 74 (**D**) 69 (**E**) 79

20. Two cylindrical tanks sit side by side on a level surface. The first tank has a radius of 4 metres, a height of 10 metres, and is full of water. The second tank has a radius of 6 metres, a height of 8 metres, and is empty. Water is pumped from the first tank to the second until the depth of water in both tanks is the same. The depth of water in each tank, in metres, is

(**A**) 4 (**B**) 5 (**C**) $\frac{46}{15}$ (**D**) $\frac{52}{17}$ (**E**) $\frac{40}{13}$

Part C: Each correct answer is worth 8.

21. In the diagram, the circle has centre O. The shaded sector AOB has sector angle 90°, and AB has arc length 2π units. The area of sector AOB is

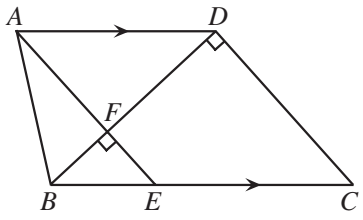


(**A**) 4π (**B**) 16π (**C**) 6π
(**D**) 24π (**E**) 8π

22. In how many ways can 75 be expressed as the sum of two or more consecutive positive integers?

(**A**) 1 (**B**) 2 (**C**) 3 (**D**) 4 (**E**) 5

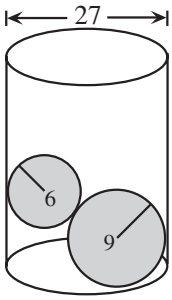
23. In trapezoid $ABCD$, AD is parallel to BC. Also, BD is perpendicular to DC. The point F is chosen on line BD so that AF is perpendicular to BD. AF is extended to meet BC at point E. If $AB = 41$, $AD = 50$ and $BF = 9$, what is the area of quadrilateral $FECD$?



(**A**) 900 (**B**) 1523.5 (**C**) 960
(**D**) 1560 (**E**) 1300

continued ...

24. A cylinder, which has a diameter of 27 and a height of 30, contains two lead spheres with radii 6 and 9, with the larger sphere sitting on the bottom of the cylinder, as shown. Water is poured into the cylinder so that it just covers both spheres. The volume of water required is



(**A**) 3672π (**B**) 3660π (**C**) 3375π
(**D**) 3114π (**E**) 4374π

25. A *lattice point* is a point (x, y) where both x and y are integers For how many different integer values of k will the two lines $kx - 5y + 7 = 0$ and $k^2x - 5y + 1 = 0$ intersect at a lattice point?

(**A**) 1 (**B**) 2 (**C**) 3 (**D**) 4 (**E**) 5

Canadian Mathematics Competition

An activity of The Centre for Education in Mathematics and Computing, University of Waterloo, Waterloo, Ontario

Cayley Contest (Grade 10)

Wednesday, February 21, 2001

Time: 1 hour

Calculators are permitted, providing they are non-programmable and without graphic displays.

Instructions

1. Do not open the contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper right corner.
5. **Be certain that you code your name, age, sex, grade, and the contest you are writing on the response form. Only those who do so can be counted as official contestants.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. When you have decided on your choice, fill in the appropriate circles on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 20.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor instructs you to begin, you will have *sixty* minutes of working time.

Scoring:	There is *no penalty* for an incorrect answer. Each unanswered question is worth 2, to a maximum of 20.

Part A: Each correct answer is worth 5.

1. The value of $\dfrac{5(6)-3(4)}{6+3}$ is

   (**A**) 1 (**B**) 2 (**C**) 6 (**D**) 12 (**E**) 31

2. When $\frac{1}{4}$ of 15 is multiplied by $\frac{1}{3}$ of 10, the answer is

   (**A**) 5 (**B**) $\frac{25}{2}$ (**C**) $\frac{85}{12}$ (**D**) $\frac{99}{8}$ (**E**) $\frac{25}{7}$

3. If $x=\frac{1}{4}$, which of the following has the largest value?

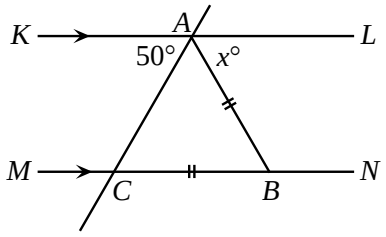
   (**A**) x (**B**) x^2 (**C**) $\frac{1}{2}x$ (**D**) $\dfrac{1}{x}$ (**E**) $\sqrt{x}$

4. In a school, 30 boys and 20 girls entered the Cayley competition. Certificates were awarded to 10% of the boys and 20% of the girls. Of the students who participated, the percentage that received certificates was

   (**A**) 14 (**B**) 15 (**C**) 16 (**D**) 30 (**E**) 50

5. In the diagram, KL is parallel to MN, $AB=BC$, and $\angle KAC=50°$. The value of x is

   (**A**) 40 (**B**) 65 (**C**) 25
   (**D**) 100 (**E**) 80

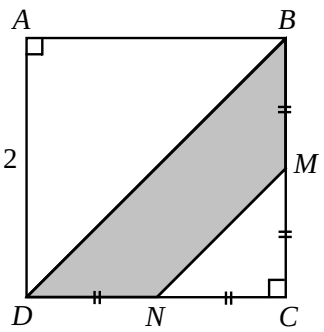


6. Dean scored a total of 252 points in 28 basketball games. Ruth played 10 fewer games than Dean. Her scoring average was 0.5 points per game higher than Dean's scoring average. How many points, in total, did Ruth score?

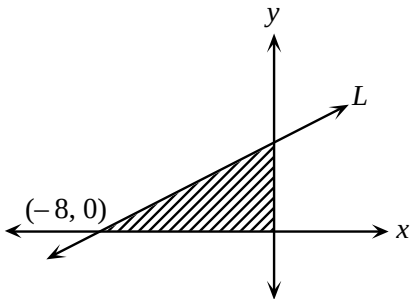
   (**A**) 153 (**B**) 171 (**C**) 180 (**D**) 266 (**E**) 144

7. In the diagram, square $ABCD$ has side length 2, with M the midpoint of BC and N the midpoint of CD. The area of the shaded region $BMND$ is

   (**A**) 1 (**B**) $2\sqrt{2}$ (**C**) $\frac{4}{3}$
   (**D**) $\frac{3}{2}$ (**E**) $4-\frac{3}{2}\sqrt{2}$



8. The line L crosses the x-axis at $(-8, 0)$. The area of the shaded region is 16. What is the slope of the line L?



(**A**) $\frac{1}{2}$ (**B**) 4 (**C**) $-\frac{1}{2}$
(**D**) 2 (**E**) -2

9. If $\left[\left(10^3\right)\left(10^x\right)\right]^2 = 10^{18}$, the value of x is

(**A**) $\sqrt{2}$ (**B**) 12 (**C**) 6 (**D**) 1 (**E**) 3

10. The sum of five consecutive integers is 75. The sum of the largest and smallest of these five integers is

(**A**) 15 (**B**) 25 (**C**) 26 (**D**) 30 (**E**) 32

Part B: Each correct answer is worth 6.

11. When a positive integer N is divided by 60, the remainder is 49. When N is divided by 15, the remainder is

(**A**) 0 (**B**) 3 (**C**) 4 (**D**) 5 (**E**) 8

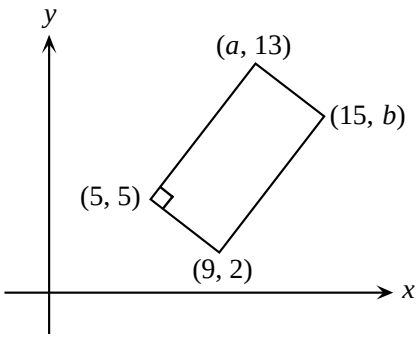
12. The 6 members of an executive committee want to call a meeting. Each of them phones 6 different people, who in turn each calls 6 other people. If no one is called more than once, how many people will know about the meeting?

(**A**) 18 (**B**) 36 (**C**) 216 (**D**) 252 (**E**) 258

13. The sequences 3, 20, 37, 54, 71, … and 16, 27, 38, 49, 60, 71, … each have 71 as a common term. The next term that these sequences have in common is

(**A**) 115 (**B**) 187 (**C**) 258 (**D**) 445 (**E**) 1006

14. In the rectangle shown, the value of $a - b$ is



(**A**) -3 (**B**) -1 (**C**) 0
(**D**) 3 (**E**) 1

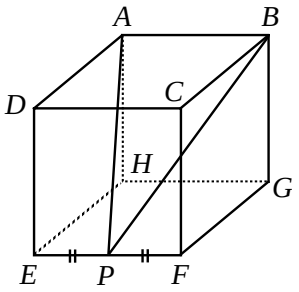
15. A small island has $\frac{2}{5}$ of its surface covered by forest and $\frac{1}{4}$ of the remainder of its surface by sand dunes. The island also has 90 hectares covered by farm land. If the island is made up of only forest, sand dunes and farm land, what is the total area of the island, to the nearest hectare?

(**A**) 163 (**B**) 120 (**C**) 200 (**D**) 138 (**E**) 257

16. How many integer values of x satisfy $\frac{x-1}{3} < \frac{5}{7} < \frac{x+4}{5}$?

(**A**) 0 (**B**) 1 (**C**) 2 (**D**) 3 (**E**) 4

17. $ABCDEFGH$ is a cube having a side length of 2. P is the midpoint of EF, as shown. The area of ΔAPB is



(**A**) $\sqrt{8}$ (**B**) 3 (**C**) $\sqrt{32}$
(**D**) $\sqrt{2}$ (**E**) 6

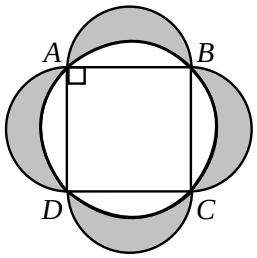
18. How many five-digit positive integers, divisible by 9, can be written using only the digits 3 and 6?

(**A**) 5 (**B**) 2 (**C**) 12 (**D**) 10 (**E**) 8

19. Three different numbers are chosen such that when each of the numbers is added to the average of the remaining two, the numbers 65, 69 and 76 result. The average of the three original numbers is

(**A**) 34 (**B**) 35 (**C**) 36 (**D**) 37 (**E**) 38

20. Square $ABCD$ with side length 2 is inscribed in a circle, as shown. Using each side of the square as a diameter, semi-circular arcs are drawn. The area of the shaded region outside the circle and inside the semi-circles is



(**A**) π (**B**) 4 (**C**) $2\pi - 2$
(**D**) $\pi + 1$ (**E**) $2\pi - 4$

Part C: Each correct answer is worth 8.

21. Point P is on the line $y = 5x + 3$. The coordinates of point Q are $(3, -2)$. If M is the midpoint of PQ, then M must lie on the line

(**A**) $y = \frac{5}{2}x - \frac{7}{2}$ (**B**) $y = 5x + 1$ (**C**) $y = -\frac{1}{5}x - \frac{7}{5}$ (**D**) $y = \frac{5}{2}x + \frac{1}{2}$ (**E**) $y = 5x - 7$

22. What is the shortest distance between two circles, the first having centre $A(5, 3)$ and radius 12, and the other with centre $B(2, -1)$ and radius 6?

(**A**) 1 (**B**) 2 (**C**) 3 (**D**) 4 (**E**) 5

continued ...

23. A sealed bottle, which contains water, has been constructed by attaching a cylinder of radius 1 cm to a cylinder of radius 3 cm, as shown in Figure A. When the bottle is right side up, the height of the water inside is 20 cm, as shown in the cross-section of the bottle in Figure B. When the bottle is upside down, the height of the liquid is 28 cm, as shown in Figure C. What is the total height, in cm, of the bottle?

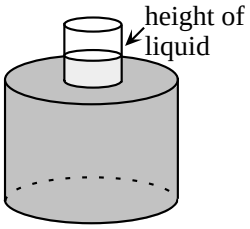


Figure A

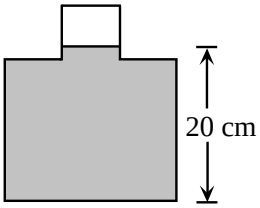


Figure B

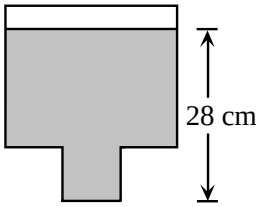


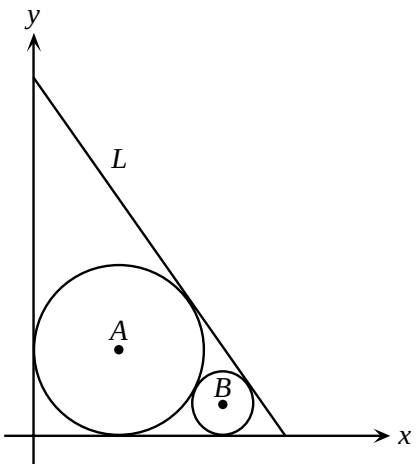
Figure C

(**A**) 29 (**B**) 30 (**C**) 31 (**D**) 32 (**E**) 48

24. A palindrome is a positive integer whose digits are the same when read forwards or backwards. For example, 2882 is a four-digit palindrome and 49194 is a five-digit palindrome. There are pairs of four-digit palindromes whose sum is a five-digit palindrome. One such pair is 2882 and 9339. How many such pairs are there?

(**A**) 28 (**B**) 32 (**C**) 36 (**D**) 40 (**E**) 44

25. The circle with centre A has radius 3 and is tangent to both the positive x-axis and positive y-axis, as shown. Also, the circle with centre B has radius 1 and is tangent to both the positive x-axis and the circle with centre A. The line L is tangent to both circles. The y-intercept of line L is



(**A**) $3+6\sqrt{3}$ (**B**) $10+3\sqrt{2}$ (**C**) $8\sqrt{3}$

(**D**) $10+2\sqrt{3}$ (**E**) $9+3\sqrt{3}$

❖ ❖ ❖ ❖ ❖

Canadian
Mathematics
Competition

An activity of The Centre for Education in Mathematics and Computing, University of Waterloo, Waterloo, Ontario

Cayley Contest (Grade 10)

Wednesday, February 23, 2000

Time: 1 hour

Calculators are permitted, providing they are non-programmable and without graphic displays.

Instructions

1. Do not open the contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper right corner.
5. **Be certain that you code your name, age, sex, grade, and the contest you are writing on the response form. Only those who do so can be counted as official contestants.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. When you have decided on your choice, fill in the appropriate circles on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 20.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor instructs you to begin, you will have *sixty* minutes of working time.

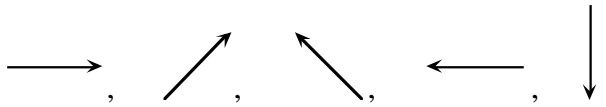
Scoring: There is *no penalty* for an incorrect answer.
Each unanswered question is worth 2 credits, to a maximum of 20 credits.

Part A: Each correct answer is worth 5.

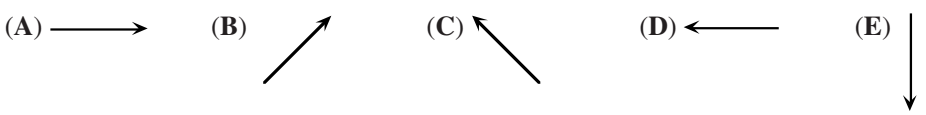
1. The value of $2(5-2)-5^2$ is

   (**A**) –19 (**B**) –4 (**C**) 1 (**D**) –11 (**E**) –17

2. If the following sequence of five arrows repeats itself continuously, what arrow would be in the 48th position?

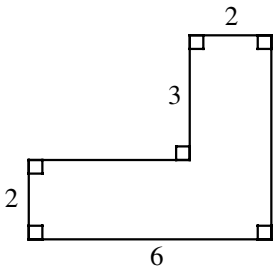


   (**A**) (**B**) (**C**) (**D**) (**E**)



3. In the given diagram, the numbers shown are the lengths of the sides. What is the perimeter of the figure?

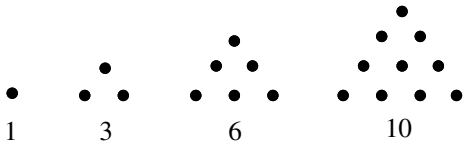
   (**A**) 13 (**B**) 18 (**C**) 22
   (**D**) 21 (**E**) 19



4. A farmer has 7 cows, 8 sheep and 6 goats. How many more goats should be bought so that half of her animals will be goats?

   (**A**) 18 (**B**) 15 (**C**) 21 (**D**) 9 (**E**) 6

5. The first four triangular numbers 1, 3, 6 and 10 are illustrated in the diagram. What is the tenth triangular number?



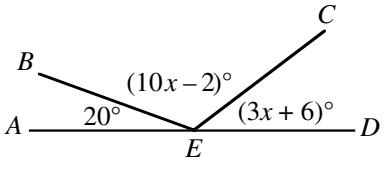
   (**A**) 55 (**B**) 45 (**C**) 66
   (**D**) 78 (**E**) 50

6. The sum of the digits of an even ten digit integer is 89. The last digit is

   (**A**) 0 (**B**) 2 (**C**) 4 (**D**) 6 (**E**) 8

7. If AD is a straight line segment and E is a point on AD, determine the measure of $\angle CED$.

   (**A**) 20° (**B**) 12° (**C**) 42°
   (**D**) 30° (**E**) 45°



8. On a 240 kilometre trip, Corey's father drove $\frac{1}{2}$ of the distance. His mother drove $\frac{3}{8}$ of the total distance and Corey drove the remaining distance. How many kilometres did Corey drive?

(**A**) 80 (**B**) 40 (**C**) 210 (**D**) 30 (**E**) 55

9. Evaluate $(-50)+(-48)+(-46)+...+54+56$.

(**A**) 156 (**B**) 10 (**C**) 56 (**D**) 110 (**E**) 162

10. The ages of three contestants in the Cayley Contest are 15 years, 9 months; 16 years, 1 month; and 15 years, 8 months. Their average (mean) age is

(**A**) 15 years, 8 months (**B**) 15 years, 9 months (**C**) 15 years, 10 months
(**D**) 15 years, 11 months (**E**) 16 years

Part B: Each correct answer is worth 6.

11. A store had a sale on T-shirts. For every two T-shirts purchased at the regular price, a third T-shirt was bought for \$1.00. Twelve T-shirts were bought for \$120.00. What was the regular price for one T-shirt?

(**A**) \$10.00 (**B**) \$13.50 (**C**) \$14.00 (**D**) \$14.50 (**E**) \$15.00

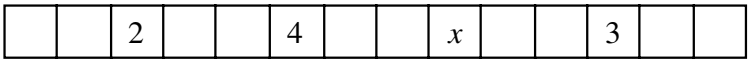
12. Natural numbers are equally spaced around a circle in order from 1 to n. If the number 5 is directly opposite the number 14, then n is

(**A**) 14 (**B**) 15 (**C**) 16 (**D**) 18 (**E**) 20

13. The average of 19 consecutive integers is 99. The largest of these integers is

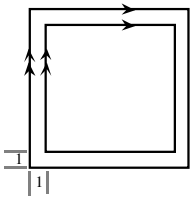
(**A**) 118 (**B**) 108 (**C**) 109 (**D**) 117 (**E**) 107

14. A positive integer is to be placed in each box. The product of any four adjacent integers is always 120. What is the value of x?



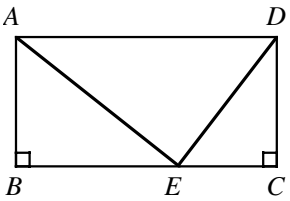
(**A**) 1 (**B**) 2 (**C**) 3 (**D**) 4 (**E**) 5

15. Eight squares with the same centre have parallel sides and are one unit apart. The two largest squares are shown. If the largest square has a perimeter of 96, what is the perimeter of the smallest square?



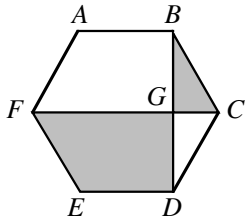
(**A**) 40 (**B**) 68 (**C**) 32
(**D**) 64 (**E**) 89

16. In the diagram, $ABCD$ is a rectangle with $AD = 13$, $DE = 5$ and $EA = 12$. The area of $ABCD$ is



(**A**) 39 (**B**) 60 (**C**) 52
(**D**) 30 (**E**) 25

17. In the regular hexagon $ABCDEF$, two of the diagonals, FC and BD, intersect at G. The ratio of the area of quadrilateral $FEDG$ to ΔBCG is

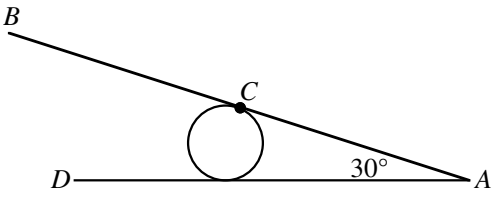


(**A**) $3\sqrt{3}:1$ (**B**) 4:1 (**C**) 6:1
(**D**) $2\sqrt{3}:1$ (**E**) 5:1

18. If a, b and c are distinct positive integers such that $abc = 16$, then the largest possible value of $a^b - b^c + c^a$ is

(**A**) 253 (**B**) 63 (**C**) 249 (**D**) 263 (**E**) 259

19. A metal rod with ends A and B is welded at its middle, C, to a cylindrical drum of diameter 12. The rod touches the ground at A making a $30°$ angle. The drum starts to roll along AD in the direction of D. How far along AD must the drum roll for B to touch the ground?



(**A**) π (**B**) 2π (**C**) 3π
(**D**) 4π (**E**) 5π

20. Twenty pairs of integers are formed using each of the integers 1, 2, 3, ..., 40 once. The positive difference between the integers in each pair is 1 or 3. (For example, 5 can be paired with 2, 4, 6 or 8.) If the resulting differences are added together, the greatest possible sum is

(**A**) 50 (**B**) 54 (**C**) 56 (**D**) 58 (**E**) 60

Part C: Each correct answer is worth 8.

21. A wooden rectangular prism has dimensions 4 by 5 by 6. This solid is painted green and then cut into 1 by 1 by 1 cubes. The ratio of the number of cubes with exactly two green faces to the number of cubes with three green faces is

(**A**) 9:2 (**B**) 9:4 (**C**) 6:1 (**D**) 3:1 (**E**) 5:2

22. An ant walks inside a 18 cm by 150 cm rectangle. The ant's path follows straight lines which always make angles of $45°$ to the sides of the rectangle. The ant starts from a point X on one of the shorter sides. The first time the ant reaches the opposite side, it arrives at the midpoint. What is the distance, in centimetres, from X to the nearest corner of the rectangle?

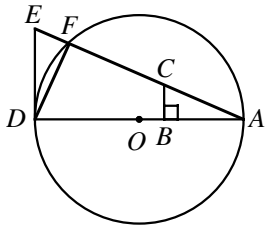
(**A**) 3 (**B**) 4 (**C**) 6 (**D**) 8 (**E**) 9

22. The left most digit of an integer of length 2000 digits is 3. In this integer, any two consecutive digits must be divisible by 17 or 23. The 2000th digit may be either 'a' or 'b'. What is the value of $a + b$?

(**A**) 3 (**B**) 7 (**C**) 4 (**D**) 10 (**E**) 17

continued ...

24. In the diagram shown, $\angle ABC = 90°$, $CB \parallel ED$, $AB = DF$, $AD = 24$, $AE = 25$ and O is the centre of the circle. Determine the perimeter of $CBDF$.



(**A**) 39 (**B**) 40 (**C**) 42
(**D**) 43 (**E**) 44

25. For the system of equations $x^2 + x^2y^2 + x^2y^4 = 525$ and $x + xy + xy^2 = 35$, the sum of the real y values that satisfy the equations is

(**A**) 20 (**B**) 2 (**C**) $\frac{3}{2}$ (**D**) $\frac{55}{2}$ (**E**) $\frac{5}{2}$

Canadian Mathematics Competition

An activity of The Centre for Education in Mathematics and Computing, University of Waterloo, Waterloo, Ontario

Cayley Contest (Grade 10)

Wednesday, February 24, 1999

C.M.C. Sponsors:

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Time: 1 hour

Calculators are permitted, providing they are non-programmable and without graphic displays.

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   Each unanswered question is worth 2 credits, to a maximum of 20 credits.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor instructs you to begin, you will have *sixty* minutes of working time.

Scoring: There is *no penalty* for an incorrect answer.
Each unanswered question is worth 2 credits, to a maximum of 20 credits.

Part A: Each question is worth 5 credits.

1. The value of $3^2 + 7^2 - 5^2$ is

   (**A**) 75 (**B**) 83 (**C**) 33 (**D**) 25 (**E**) 10

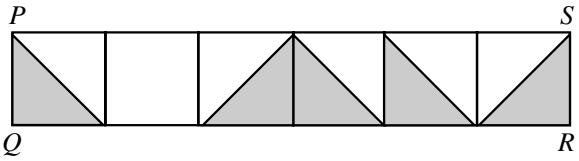
2. If 8 is added to the square of 5 the result is divisible by

   (**A**) 5 (**B**) 2 (**C**) 8 (**D**) 23 (**E**) 11

3. Today is Wednesday. What day of the week will it be 100 days from now?

   (**A**) Monday (**B**) Tuesday (**C**) Thursday (**D**) Friday (**E**) Saturday

4. The rectangle $PQRS$ is divided into six equal squares and shaded as shown. What fraction of $PQRS$ is shaded?



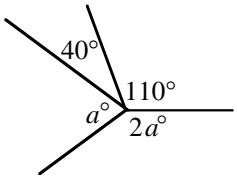
   (**A**) $\frac{1}{2}$ (**B**) $\frac{7}{12}$ (**C**) $\frac{5}{11}$ (**D**) $\frac{6}{11}$ (**E**) $\frac{5}{12}$

5. If $x = 4$ and $y = 3x$ and $z = 2y$, then the value of $y + z$ is

   (**A**) 12 (**B**) 20 (**C**) 40 (**D**) 24 (**E**) 36

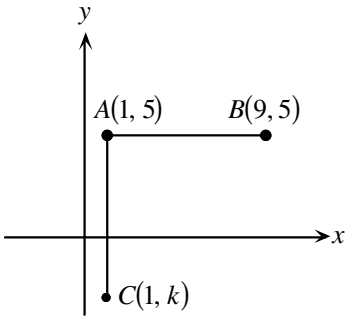
6. In the diagram, the value of a is

   (**A**) 50 (**B**) 65 (**C**) 70
   (**D**) 105 (**E**) 110

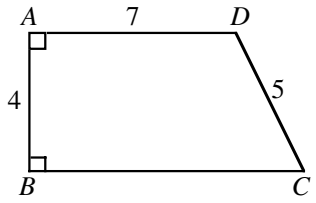


7. In the diagram, AB and AC have equal lengths. What is the value of k?

   (**A**) -3 (**B**) -4 (**C**) -5
   (**D**) -7 (**E**) -8



8. In the diagram, $AD < BC$. What is the perimeter of $ABCD$?

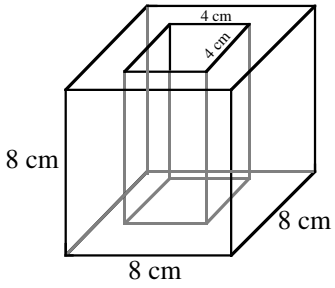


(**A**) 23 (**B**) 26 (**C**) 27
(**D**) 28 (**E**) 30

9. Three CD's are bought at an average cost of \$15 each. If a fourth CD is purchased, the average cost becomes \$16. What is the cost of the fourth CD?

(**A**) \$16 (**B**) \$17 (**C**) \$18 (**D**) \$19 (**E**) \$20

10. An 8 cm cube has a 4 cm square hole cut through its centre, as shown. What is the remaining volume, in cm^3?



(**A**) 64 (**B**) 128 (**C**) 256
(**D**) 384 (**E**) 448

Part B: Each question is worth 6 credits.

11. The time on a digital clock is 5:55. How many minutes will pass before the clock next shows a time with all digits identical?

(**A**) 71 (**B**) 72 (**C**) 255 (**D**) 316 (**E**) 436

12. The numbers 49, 29, 9, 40, 22, 15, 53, 33, 13, 47 are grouped in pairs so that the sum of each pair is the same. Which number is paired with 15?

(**A**) 33 (**B**) 40 (**C**) 47 (**D**) 49 (**E**) 53

13. The units digit in the product $\left(5^2+1\right)\left(5^3+1\right)\left(5^{23}+1\right)$ is

(**A**) 0 (**B**) 1 (**C**) 2 (**D**) 5 (**E**) 6

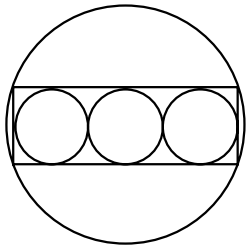
14. In an election for class president, 61 votes are cast by students who are voting to choose one of four candidates. Each student must vote for only one candidate. The candidate with the highest number of votes is the winner. The smallest number of votes the winner can receive is

(**A**) 15 (**B**) 16 (**C**) 21 (**D**) 30 (**E**) 31

15. A chocolate drink is 6% pure chocolate, by volume. If 10 litres of pure milk are added to 50 litres of this drink, the percent of chocolate in the new drink is

(**A**) 5 (**B**) 16 (**C**) 10 (**D**) 3 (**E**) 26

16. Three circles, each with a radius of 10 cm, are drawn tangent to each other so that their centres are all in a straight line. These circles are inscribed in a rectangle which is inscribed in another circle. The area of the largest circle is

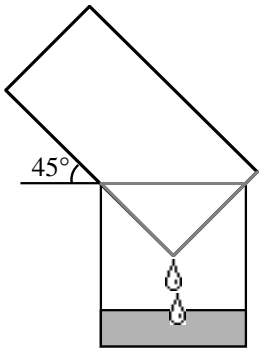


(**A**) 1000π (**B**) 1700π (**C**) 900π
(**D**) 1600π (**E**) 1300π

17. Let N be the smallest positive integer whose digits have a product of 2000. The sum of the digits of N is

(**A**) 21 (**B**) 23 (**C**) 25 (**D**) 27 (**E**) 29

18. A cylindrical pail containing water drains into a cylindrical tub 40 cm across and 50 cm deep, while resting at an angle of $45°$ to the horizontal, as shown. How deep is the water in the tub when its level reaches the pail?

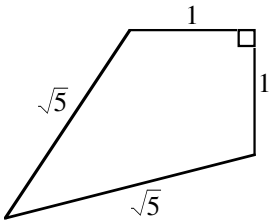


(**A**) 10 cm (**B**) 20 cm (**C**) 30 cm
(**D**) 35 cm (**E**) 40 cm

19. A number is *Beprisque* if it is the only natural number between a prime number and a perfect square (e.g. 10 is Beprisque but 12 is not). The number of *two-digit* Beprisque numbers (including 10) is

(**A**) 1 (**B**) 2 (**C**) 3 (**D**) 4 (**E**) 5

20. The area of the given quadrilateral is



(**A**) $\frac{3}{2}$ (**B**) $\sqrt{5}$ (**C**) $\frac{1+\sqrt{10}}{2}$
(**D**) 2 (**E**) 3

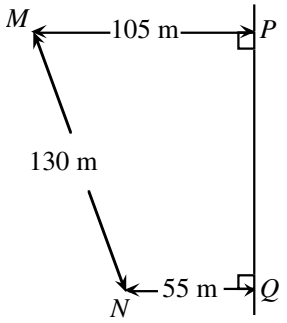
Part C: Each question is worth 8 credits.

21. A number is formed using the digits 1, 2, ..., 9. Any digit can be used more than once, but adjacent digits cannot be the same. Once a pair of adjacent digits has occurred, that pair, in that order, cannot be used again. How many digits are in the largest such number?

(**A**) 72 (**B**) 73 (**C**) 144 (**D**) 145 (**E**) 91

continued ...

22. A main gas line runs through P and Q. From some point T on PQ, a supply line runs to a house at point M. A second supply line from T runs to a house at point N. What is the minimum total length of pipe required for the two supply lines?



(**A**) 200 (**B**) 202 (**C**) 198
(**D**) 210 (**E**) 214

23. How many integers can be expressed as a sum of three distinct numbers chosen from the set $\{4, 7, 10, 13, ..., 46\}$?

(**A**) 45 (**B**) 37 (**C**) 36 (**D**) 43 (**E**) 42

24. The sum of all values of x that satisfy the equation $\left(x^2 - 5x + 5\right)^{x^2+4x-60} = 1$ is

(**A**) -4 (**B**) 3 (**C**) 1 (**D**) 5 (**E**) 6

25. If $a = 3^p$, $b = 3^q$, $c = 3^r$, and $d = 3^s$ and if p, q, r, and s are positive integers, determine the smallest value of $p + q + r + s$ such that $a^2 + b^3 + c^5 = d^7$.

(**A**) 17 (**B**) 31 (**C**) 106 (**D**) 247 (**E**) 353

Canadian Mathematics Competition

An activity of The Centre for Education in Mathematics and Computing, University of Waterloo, Waterloo, Ontario

Cayley Contest (Grade 10)

Wednesday, February 18, 1998

Time: 1 hour

Calculators are permitted, providing they are non-programmable and without graphic displays.

Instructions

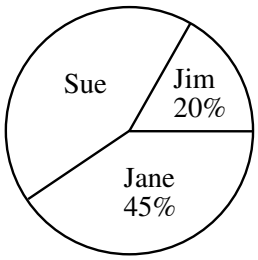
1. Do not open the contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper right corner.
5. **Be certain that you code your name, age, sex, grade, and the contest you are writing on the response form. Only those who do so can be counted as official contestants.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. When you have decided on your choice, fill in the appropriate circles on the response form.
7. Scoring: Each correct answer is worth 5 credits in Part A, 6 credits in Part B, and 8 credits in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2 credits, to a maximum of 20 credits.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor instructs you to begin, you will have *sixty* minutes of working time.

Scoring: There is *no penalty* for an incorrect answer.
Each unanswered question is worth 2 credits, to a maximum of 20 credits.

Part A: Each question is worth 5 credits.

1. The value of $(0.3)^2 + 0.1$ is
(**A**) 0.7 (**B**) 1 (**C**) 0.1 (**D**) 0.19 (**E**) 0.109

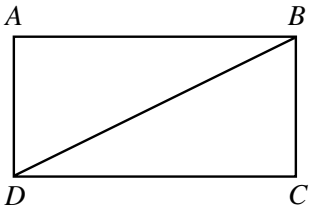
2. The pie chart shows a percentage breakdown of 1000 votes in a student election. How many votes did Sue receive?
(**A**) 550 (**B**) 350 (**C**) 330
(**D**) 450 (**E**) 935



3. The expression $\dfrac{a^9 \times a^{15}}{a^3}$ is equal to
(**A**) a^{45} (**B**) a^8 (**C**) a^{18} (**D**) a^{14} (**E**) a^{21}

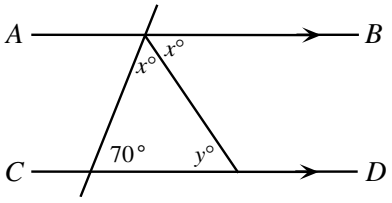
4. The product of two positive integers p and q is 100. What is the largest possible value of $p+q$?
(**A**) 52 (**B**) 101 (**C**) 20 (**D**) 29 (**E**) 25

5. In the diagram, $ABCD$ is a rectangle with $DC = 12$. If the area of triangle BDC is 30, what is the perimeter of rectangle $ABCD$?
(**A**) 34 (**B**) 44 (**C**) 30
(**D**) 29 (**E**) 60



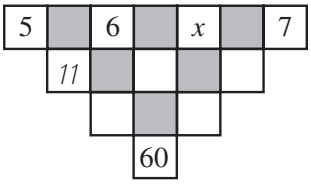
6. If $x = 2$ is a solution of the equation $qx - 3 = 11$, the value of q is
(**A**) 4 (**B**) 7 (**C**) 14 (**D**) -7 (**E**) -4

7. In the diagram, AB is parallel to CD. What is the value of y?
(**A**) 75 (**B**) 40 (**C**) 35
(**D**) 55 (**E**) 50



8. The vertices of a triangle have coordinates $(1,1)$, $(7,1)$ and $(5,3)$. What is the area of this triangle?
(**A**) 12 (**B**) 8 (**C**) 6 (**D**) 7 (**E**) 9

9. The number in an unshaded square is obtained by adding the numbers connected to it from the row above. (The '*11*' is one such number.) The value of x must be
(**A**) 4 (**B**) 6 (**C**) 9
(**D**) 15 (**E**) 10



10. The sum of the digits of a five-digit positive integer is 2. (A five-digit integer cannot start with zero.) The number of such integers is

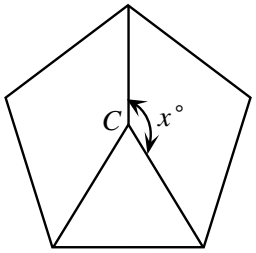
(**A**) 1 (**B**) 2 (**C**) 3 (**D**) 4 (**E**) 5

Part B: Each question is worth 6 credits.

11. If $x+y+z=25$, $x+y=19$ and $y+z=18$, then y equals

(**A**) 13 (**B**) 17 (**C**) 12 (**D**) 6 (**E**) -6

12. A regular pentagon with centre C is shown. The value of x is



(**A**) 144 (**B**) 150 (**C**) 120
(**D**) 108 (**E**) 72

13. If the surface area of a cube is 54, what is its volume?

(**A**) 36 (**B**) 9 (**C**) $\frac{81\sqrt{3}}{8}$ (**D**) 27 (**E**) $162\sqrt{6}$

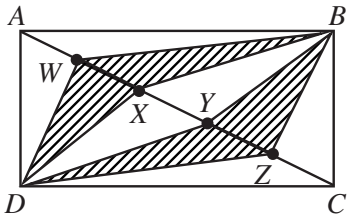
14. The number of solutions (x, y) of the equation $3x+y=100$, where x and y are positive integers, is

(**A**) 33 (**B**) 35 (**C**) 100 (**D**) 101 (**E**) 97

15. If $\sqrt{y-5}=5$ and $2^x=8$, then $x+y$ equals

(**A**) 13 (**B**) 28 (**C**) 33 (**D**) 35 (**E**) 38

16. Rectangle $ABCD$ has length 9 and width 5. Diagonal AC is divided into 5 equal parts at W, X, Y, and Z. Determine the area of the shaded region.



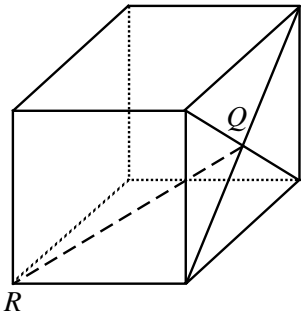
(**A**) 36 (**B**) $\frac{36}{5}$ (**C**) 18

(**D**) $\frac{4\sqrt{106}}{5}$ (**E**) $\frac{2\sqrt{106}}{5}$

17. If $N=\left(7^{p+4}\right)\left(5^q\right)\left(2^3\right)$ is a perfect cube, where p and q are positive integers, the smallest possible value of $p+q$ is

(**A**) 5 (**B**) 2 (**C**) 8 (**D**) 6 (**E**) 12

18. Q is the point of intersection of the diagonals of one face of a cube whose edges have length 2 units. The length of QR is

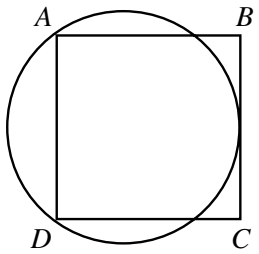


(**A**) 2 (**B**) $\sqrt{8}$ (**C**) $\sqrt{5}$
(**D**) $\sqrt{12}$ (**E**) $\sqrt{6}$

19. Mr. Anderson has more than 25 students in his class. He has more than 2 but fewer than 10 boys and more than 14 but fewer than 23 girls in his class. How many different class *sizes* would satisfy these conditions?

(**A**) 5 (**B**) 6 (**C**) 7 (**D**) 3 (**E**) 4

20. Each side of square $ABCD$ is 8. A circle is drawn through A and D so that it is tangent to BC. What is the radius of this circle?



(**A**) 4 (**B**) 5 (**C**) 6

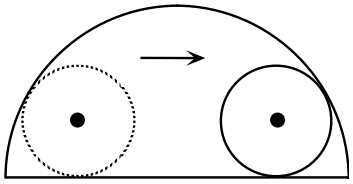
(**D**) $4\sqrt{2}$ (**E**) 5.25

Part C: Each question is worth 8 credits.

21. When Betty substitutes $x = 1$ into the expression $ax^3 - 2x + c$ its value is -5. When she substitutes $x = 4$ the expression has value 52. One value of x that makes the expression equal to zero is

(**A**) 2 (**B**) $\frac{5}{2}$ (**C**) 3 (**D**) $\frac{7}{2}$ (**E**) 4

22. A wheel of radius 8 rolls along the diameter of a semicircle of radius 25 until it bumps into this semicircle. What is the length of the portion of the diameter that cannot be touched by the wheel?



(**A**) 8 (**B**) 12 (**C**) 15

(**D**) 17 (**E**) 20

23. There are four unequal, positive integers a, b, c, and N such that $N = 5a + 3b + 5c$. It is also true that $N = 4a + 5b + 4c$ and N is between 131 and 150. What is the value of $a + b + c$?

(**A**) 13 (**B**) 17 (**C**) 22 (**D**) 33 (**E**) 36

24. Three rugs have a combined area of 200 m^2. By overlapping the rugs to cover a floor area of 140 m^2, the area which is covered by exactly two layers of rug is 24 m^2. What area of floor is covered by three layers of rug?

(**A**) 12 m^2 (**B**) 18 m^2 (**C**) 24 m^2 (**D**) 36 m^2 (**E**) 42 m^2

25. One way to pack a 100 by 100 square with 10 000 circles, each of diameter 1, is to put them in 100 rows with 100 circles in each row. If the circles are repacked so that the centres of any three tangent circles form an equilateral triangle, what is the maximum number of additional circles that can be packed?

(**A**) 647 (**B**) 1442 (**C**) 1343 (**D**) 1443 (**E**) 1344

Canadian Mathematics Competition

An activity of The Centre for Education in Mathematics and Computing, University of Waterloo, Waterloo, Ontario

Cayley Contest (Grade 10)

Wednesday, February 19, 1997

Part A: Each question is worth 5 credits.

1. The value of $2\frac{1}{10}+3\frac{11}{100}$ is

   (**A**) 5.11 (**B**) 5.111 (**C**) 5.12 (**D**) 5.21 (**E**) 5.3

2. The value of $(1)^{10}+(-1)^{8}+(-1)^{7}+(1)^{5}$ is

   (**A**) 0 (**B**) 1 (**C**) 2 (**D**) 16 (**E**) 4

3. An integer is multiplied by 2 and the result is then multiplied by 5. The final result could be

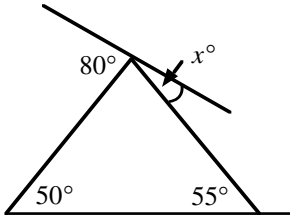
   (**A**) 64 (**B**) 32 (**C**) 12 (**D**) 25 (**E**) 30

4. The greatest number of Mondays that can occur in 45 consecutive days is

   (**A**) 5 (**B**) 6 (**C**) 7 (**D**) 8 (**E**) 9

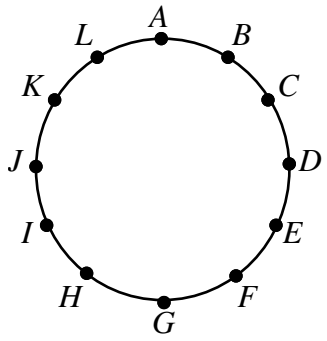
5. The value of x is

   (**A**) 25 (**B**) 30 (**C**) 50
   (**D**) 55 (**E**) 20



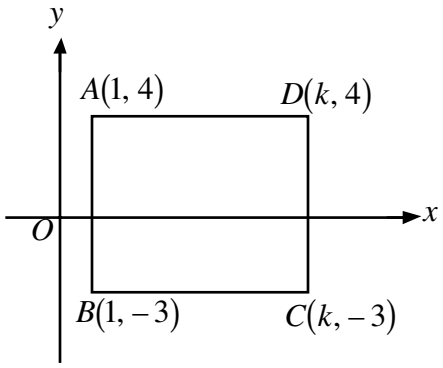
6. Twelve balloons are arranged in a circle as shown. Counting clockwise, every third balloon is popped, with C the first one popped. This process continues around the circle until two unpopped balloons remain. The last two remaining balloons are

   (**A**) B, H (**B**) B, G (**C**) A, E
   (**D**) E, J (**E**) F, K



7. In the diagram, rectangle $ABCD$ has area 70 and k is positive. The value of k is

   (**A**) 8 (**B**) 9 (**C**) 10
   (**D**) 11 (**E**) 12



8. If p, q, r, s, and t are numbers such that $r < s$, $t > q$, $q > p$, and $t < r$, which of these numbers is greatest?
(**A**) t (**B**) s (**C**) r (**D**) q (**E**) p

9. The sum of seven consecutive integers is 77. The smallest of these integers is
(**A**) 5 (**B**) 7 (**C**) 8 (**D**) 11 (**E**) 14

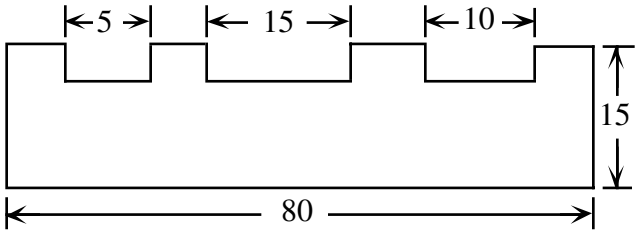
10. Each of the numbers 1, 2, 3, and 4 is assigned, in some order, to p, q, r, and s. The largest possible value of $p^q + r^s$ is
(**A**) 12 (**B**) 19 (**C**) 66 (**D**) 82 (**E**) 83

Part B: Each question is worth 6 credits.

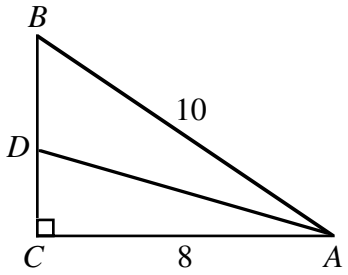
11. In the chart, the products of the numbers represented by the letters in each of the rows and columns are given. For example, $xy = 6$ and $xz = 12$. If x, y, z, and w are integers, what is the value of xw?
(**A**) 150 (**B**) 300 (**C**) 31
(**D**) 75 (**E**) 30

x	y	6
z	w	50
12	25	

12. Three small rectangles, of the same depth, are cut from a rectangular sheet of metal. The area of the remaining piece is 990. What is the depth of each cut?
(**A**) 8 (**B**) 7 (**C**) 6
(**D**) 5 (**E**) 4



13. Triangle ABC is right-angled with $AB = 10$ and $AC = 8$. If $BC = 3DC$, then AD equals
(**A**) 9 (**B**) $\sqrt{65}$ (**C**) $\sqrt{80}$
(**D**) $\sqrt{73}$ (**E**) $\sqrt{68}$



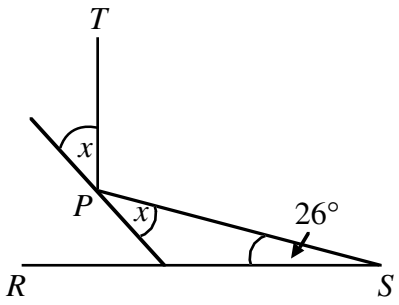
14. The digits 1, 2, 3, 4 can be arranged to form twenty-four different four digit numbers. If these twenty-four numbers are then listed from smallest to largest, in what position is 3142?
(**A**) 13th (**B**) 14th (**C**) 15th (**D**) 16th (**E**) 17th

15. The product of 20^{50} and 50^{20} is written as an integer in expanded form. The number of zeros at the end of the resulting integer is

(**A**) 70 (**B**) 71 (**C**) 90 (**D**) 140 (**E**) 210

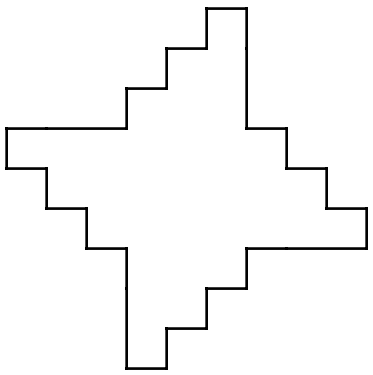
16. A beam of light shines from point S, reflects off a reflector at point P, and reaches point T so that PT is perpendicular to RS. Then x is

(**A**) 32° (**B**) 37° (**C**) 45°
(**D**) 26° (**E**) 38°



17. In the diagram adjacent edges are at right angles. The four longer edges are equal in length, and all of the shorter edges are also equal in length. The area of the shape is 528. What is the perimeter?

(**A**) 132 (**B**) 264 (**C**) 92
(**D**) 72 (**E**) 144



18. If $\frac{30}{7} = x + \frac{1}{y + \frac{1}{z}}$, where x, y, and z are positive integers, then what is the value of $x + y + z$?

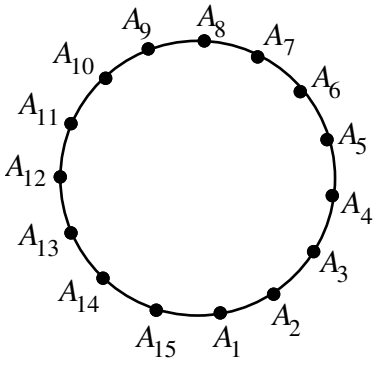
(**A**) 13 (**B**) 9 (**C**) 11 (**D**) 37 (**E**) 30

19. If $x^2yz^3 = 7^4$ and $xy^2 = 7^5$, then xyz equals

(**A**) 7 (**B**) 7^2 (**C**) 7^3 (**D**) 7^8 (**E**) 7^9

20. On a circle, fifteen points $A_1, A_2, A_3, \ldots, A_{15}$ are equally spaced. What is the size of angle $A_1A_3A_7$?

(**A**) 96° (**B**) 100° (**C**) 104°
(**D**) 108° (**E**) 120°

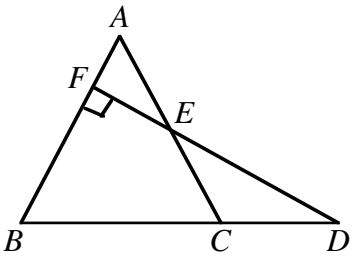


Part C: Each question is worth 8 credits.

21. If $\dfrac{\left(\frac{a}{c}+\frac{a}{b}+1\right)}{\left(\frac{b}{a}+\frac{b}{c}+1\right)}=11$, where a, b, and c are positive integers, the number of different ordered triples (a,b,c) such that $a+2b+c\le 40$ is

(**A**) 33 (**B**) 37 (**C**) 40 (**D**) 42 (**E**) 45

22. In the diagram, ΔABC is equilateral, $BC=2CD$, $AF=6$, and DEF is perpendicular to AB. What is the area of quadrilateral $FBCE$?



(**A**) $144\sqrt{3}$ (**B**) $138\sqrt{3}$ (**C**) $126\sqrt{3}$
(**D**) $108\sqrt{3}$ (**E**) $66\sqrt{3}$

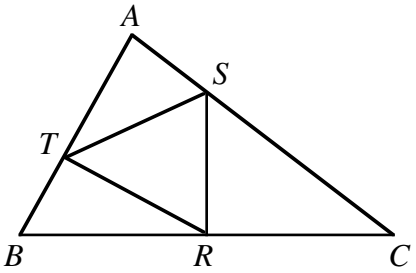
23. Given the set $\{1,2,3,5,8,13,21,34,55\}$, how many integers between 3 and 89 cannot be written as the sum of exactly two elements of the set?

(**A**) 51 (**B**) 57 (**C**) 55 (**D**) 34 (**E**) 43

24. In a convex polygon, exactly five of the interior angles are obtuse. The largest possible number of sides for this polygon is

(**A**) 7 (**B**) 8 (**C**) 9 (**D**) 10 (**E**) 11

25. In triangle ABC, $BR=RC$, $CS=3SA$, and $\dfrac{AT}{TB}=\dfrac{p}{q}$. If the area of ΔRST is twice the area of ΔTBR, then $\dfrac{p}{q}$ is equal to



(**A**) $\frac{2}{1}$ (**B**) $\frac{8}{3}$ (**C**) $\frac{5}{2}$
(**D**) $\frac{7}{4}$ (**E**) $\frac{7}{3}$