The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
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Fermat Contest

(Grade 11)

Thursday, February 24, 2011

STRONGER COMMUNITIES TOGETHER™

Time: 60 minutes
Calculators are permitted

Instructions

1. Do not open the Contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper left corner.
5. **Be certain that you code your name, age, sex, grade, and the Contest you are writing in the response form. Only those who do so can be counted as official contestants.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor tells you to begin, you will have *sixty* minutes of working time.

The names of some top-scoring students will be published in the PCF Results on our Web site, http://www.cemc.uwaterloo.ca.

Scoring:	There is *no penalty* for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. $\dfrac{2+3\times 6}{23+6}$ is equal to

(A) 1 **(B)** $\frac{11}{29}$ **(C)** $\frac{36}{29}$ **(D)** $\frac{20}{29}$ **(E)** $\frac{5}{23}$

2. If $y = 77$, then $\dfrac{7y+77}{77}$ is equal to

(A) 8 **(B)** 12 **(C)** 78 **(D)** 84 **(E)** 540

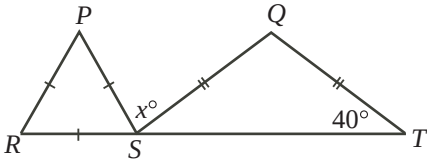
3. The area of the rectangle shown is 192. What is the perimeter of the rectangle?

(A) 64 **(B)** 384 **(C)** 192 **(D)** 1728 **(E)** 32

4. If $\sqrt{n+9} = 25$, then n equals

(A) 256 **(B)** -4 **(C)** 484 **(D)** 616 **(E)** 16

5. In the diagram, S lies on RT, $\angle QTS = 40°$, $QS = QT$, and $\triangle PRS$ is equilateral. The value of x is



(A) 50 **(B)** 60 **(C)** 80 **(D)** 90 **(E)** 100

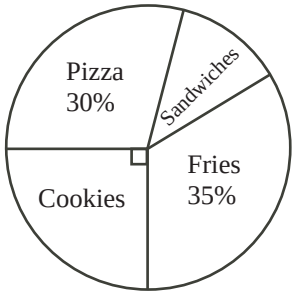
6. When three consecutive integers are added, the total is 27. When the same three integers are multiplied, the result is

(A) 504 **(B)** 81 **(C)** 720 **(D)** 729 **(E)** 990

7. The number halfway between $\frac{1}{12}$ and $\frac{1}{10}$ is

(A) $\frac{1}{11}$ **(B)** $\frac{1}{120}$ **(C)** $\frac{11}{60}$ **(D)** $\frac{11}{120}$ **(E)** $\frac{1}{22}$

8. The circle graph shown illustrates the results of a survey taken by the Fermat H.S. Student Council to determine the favourite cafeteria food. How many of the 200 students surveyed said that their favourite food was sandwiches?

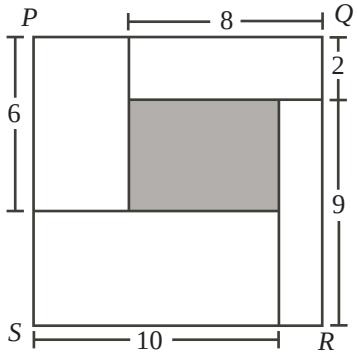


(A) 10 **(B)** 20 **(C)** 35 **(D)** 50 **(E)** 70

9. The set $S = \{1, 2, 3, \ldots, 49, 50\}$ contains the first 50 positive integers. After the multiples of 2 and the multiples of 3 are removed, how many numbers remain in the set S?

(A) 8 **(B)** 9 **(C)** 16 **(D)** 17 **(E)** 18

10. In the diagram, $PQRS$ is a square. Square $PQRS$ is divided into five rectangles, as shown. The area of the shaded rectangle is



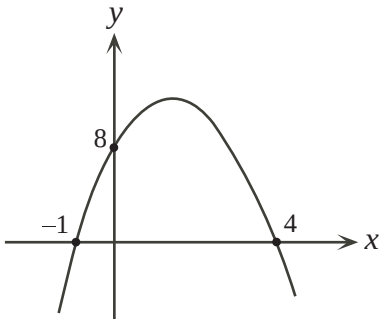
(A) 49 **(B)** 28 **(C)** 22 **(D)** 57 **(E)** 16

Part B: Each correct answer is worth 6.

11. A gumball machine that randomly dispenses one gumball at a time contains 13 red, 5 blue, 1 white, and 9 green gumballs. What is the least number of gumballs that Wally must buy to *guarantee* that he receives 3 gumballs of the same colour?

(A) 6 **(B)** 9 **(C)** 4 **(D)** 7 **(E)** 8

12. In the diagram, the parabola has x-intercepts -1 and 4, and y-intercept 8. If the parabola passes through the point $(3, w)$, what is the value of w?



(A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

13. Xavier, Yolanda, and Zixuan have a total of \$50. The ratio of the amount Xavier has to the total amount Yolanda and Zixuan have is 3 : 2. Yolanda has \$4 more than Zixuan. How much does Zixuan have?

(A) \$16 **(B)** \$8 **(C)** \$14 **(D)** \$13 **(E)** \$30

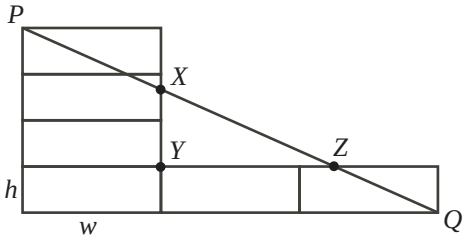
14. Which of the following *must* be an even integer?

(A) The average of two even integers
(B) The average of two prime numbers
(C) The average of two perfect squares
(D) The average of two multiples of 4
(E) The average of three consecutive integers

15. If m and n are consecutive positive integers and $n^2 - m^2 > 20$, then the minimum possible value of $n^2 + m^2$ is

(A) 29 **(B)** 181 **(C)** 265 **(D)** 23 **(E)** 221

16. Six identical rectangles with height h and width w are arranged as shown. Line segment PQ intersects the vertical side of one rectangle at X and the horizontal side of another rectangle at Z. If right-angled $\triangle XYZ$ has $YZ = 2XY$, then $\dfrac{h}{w}$ equals

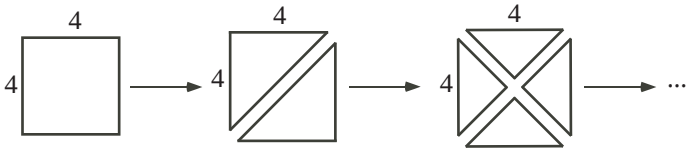


(A) $\frac{2}{3}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{3}{8}$
(D) $\frac{1}{3}$ **(E)** $\frac{3}{4}$

17. If $3^{2x} = 64$, then 3^{-x} is equal to

(A) -32 **(B)** -8 **(C)** $\frac{1}{4096}$ **(D)** $\frac{1}{32}$ **(E)** $\frac{1}{8}$

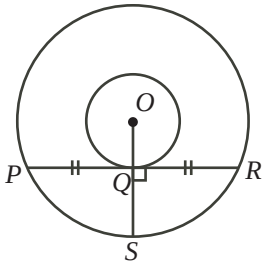
18. A 4×4 square piece of paper is cut into two identical pieces along its diagonal. The resulting triangular pieces of paper are each cut into two identical pieces.



Each of the four resulting pieces is cut into two identical pieces. Each of the eight new resulting pieces is finally cut into two identical pieces. The length of the longest edge of one of these final sixteen pieces of paper is

(A) 1 **(B)** 2 **(C)** $\frac{1}{2}$ **(D)** $\frac{1}{\sqrt{2}}$ **(E)** $2\sqrt{2}$

19. In the diagram, the two circles are centred at O. Point S is on the larger circle. Point Q is the point of intersection of OS and the smaller circle. Line segment PR is a chord of the larger circle and touches (that is, is tangent to) the smaller circle at Q. Note that OS is the perpendicular bisector of PR. If $PR = 12$ and $QS = 4$, then the radius of the larger circle is



(A) 6.0 **(B)** 5.0 **(C)** 6.5
(D) 7.2 **(E)** 20.0

20. Three real numbers a, b and c have a sum of 114 and a product of 46 656. If $b = ar$ and $c = ar^2$ for some real number r, then the value of $a + c$ is

(A) 78 **(B)** 76 **(C)** 24 **(D)** 54 **(E)** 36

Part C: Each correct answer is worth 8.

21. The positive integers are arranged in increasing order in a triangle, as shown. Each row contains one more number than the previous row. The sum of the numbers in the row that contains the number 400 is

$$\begin{array}{ccccccccccc} & & & & 1 & & & & & & \\ & & & 2 & & & 3 & & & & \\ & & 4 & & & 5 & & & 6 & & \\ & 7 & & & 8 & & & 9 & & 10 & \\ 11 & & & 12 & & & \cdots & & & & \end{array}$$

(A) 10 990 **(B)** 12 209 **(C)** 9855
(D) 10 976 **(E)** 11 368

22. The number of pairs of positive integers (p, q), with $p + q \leq 100$, that satisfy the equation $\dfrac{p + q^{-1}}{p^{-1} + q} = 17$ is

(A) 0 **(B)** 1 **(C)** 2 **(D)** 4 **(E)** 5

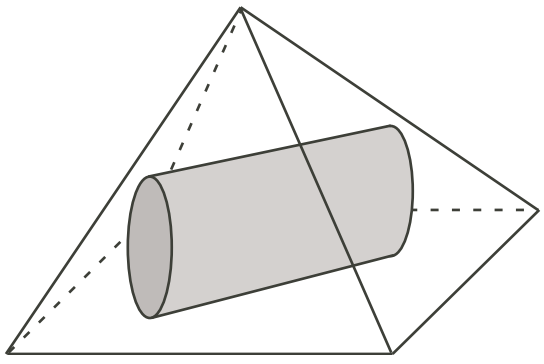
23. Dolly, Molly and Polly each can walk at 6 km/h. Their one motorcycle, which travels at 90 km/h, can accommodate at most two of them at once (and cannot drive by itself!). Let t hours be the time taken for all three of them to reach a point 135 km away. Ignoring the time required to start, stop or change directions, what is true about the smallest possible value of t?

(A) $t < 3.9$ **(B)** $3.9 \leq t < 4.1$ **(C)** $4.1 \leq t < 4.3$
(D) $4.3 \leq t < 4.5$ **(E)** $4.5 \leq t$

24. Four numbers w, x, y, z satisfy $w < x < y < z$. Each of the six possible pairs of distinct numbers has a different sum. The four smallest sums are $1, 2, 3$, and 4. What is the sum of all possible values of z?

(A) 4 **(B)** $\frac{13}{2}$ **(C)** $\frac{17}{2}$ **(D)** $\frac{15}{2}$ **(E)** 7

25. A pyramid has a square base with side length 20. A right circular cylinder has a diameter of 10 and a length of 10. The cylinder is lying on its side, completely inside the pyramid. The central axis of the cylinder lies parallel to and directly above a diagonal of the pyramid's base. The midpoint of the central axis lies directly above the centre of the square base of the pyramid.



The smallest possible height of the pyramid is closest to

(A) 15.3 **(B)** 22.1 **(C)** 21.9 **(D)** 21.7 **(E)** 15.5

2011 Fermat Contest (English)

The CENTRE for EDUCATION in MATHEMATICS and COMPUTING

For students...

Thank you for writing the 2011 Fermat Contest!
In 2010, more than 81 000 students around the world registered to write the Pascal, Cayley and Fermat Contests.

Encourage your teacher to register you for the Hypatia Contest which will be written on April 13, 2011.

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Visit our website to

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- Learn about our face-to-face workshops and our resources
- Find your school contest results

www.cemc.uwaterloo.ca

Canadian Mathematics Competition

An activity of the Centre for Education in Mathematics and Computing, University of Waterloo, Waterloo, Ontario

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Thursday, February 25, 2010

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Part A: Each correct answer is worth 5.

1. The value of $\frac{1}{2}+\frac{1}{2}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}$ is

   (A) 2 **(B)** $\frac{5}{13}$ **(C)** $\frac{5}{6}$ **(D)** 1 **(E)** $\frac{13}{6}$

2. The quantity "2% of 1" is equal to

   (A) $\frac{2}{100}$ **(B)** $\frac{2}{10}$ **(C)** 2 **(D)** 20 **(E)** 200

3. In the diagram, points P, Q, R, and S are arranged in order on a line segment. If $PQ = 1$, $QR = 2PQ$ and $RS = 3QR$, then the length of PS is

   (A) 7 **(B)** 6 **(C)** 9
   (D) 8 **(E)** 10

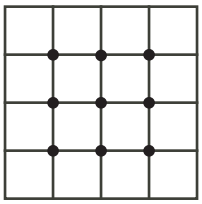
4. If $u = -6$ and $x = \frac{1}{3}(3 - 4u)$, then x equals

   (A) -23 **(B)** -7 **(C)** 9 **(D)** 2 **(E)** 25

5. If $2^x = 16$, then 2^{x+3} equals

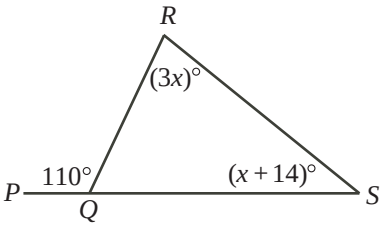
   (A) 19 **(B)** 48 **(C)** 22 **(D)** 128 **(E)** 2048

6. The nine interior intersection points on a 4 by 4 grid of squares are shown. How many interior intersection points are there on a 12 by 12 grid of squares?



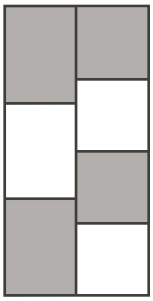
   (A) 100 **(B)** 121 **(C)** 132
   (D) 144 **(E)** 169

7. In the diagram, PQS is a straight line. What is the value of x?



   (A) 19 **(B)** 62 **(C)** 21.5
   (D) 24 **(E)** 32

8. A rectangle is divided into two vertical strips of equal width. The strip on the left is divided into three equal parts and the strip on the right is divided into four equal parts. Parts of the rectangle are then shaded as shown. What fraction of the original rectangle is shaded?



   (A) $\frac{3}{5}$ **(B)** $\frac{2}{7}$ **(C)** $\frac{4}{7}$
   (D) $\frac{7}{6}$ **(E)** $\frac{7}{12}$

9. The value of $k\nabla m$ is defined to be $k(k-m)$. For example, $7\nabla 2 = 7(7-2) = 35$. What is the value of $(5\nabla 1) + (4\nabla 1)$?

(A) 9 **(B)** 84 **(C)** 20 **(D)** 32 **(E)** 72

10. If $2x^2 = 9x - 4$ and $x \neq 4$, then the value of $2x$ is

(A) 4 **(B)** 1 **(C)** -1 **(D)** 0 **(E)** 2

Part B: Each correct answer is worth 6.

11. A loonie is a \$1 coin and a dime is a \$0.10 coin. One loonie has the same mass as 4 dimes. A bag of dimes has the same mass as a bag of loonies. The coins in the bag of loonies are worth \$400 in total. How much are the coins in the bag of dimes worth?

(A) \$40 **(B)** \$100 **(C)** \$160 **(D)** \$1000 **(E)** \$1600

12. When k candies were distributed among seven people so that each person received the same number of candies and each person received as many candies as possible, there were 3 candies left over. If instead, $3k$ candies were distributed among seven people in this way, then the number of candies left over would have been

(A) 1 **(B)** 2 **(C)** 3 **(D)** 6 **(E)** 9

13. Fifty numbers have an average of 76. Forty of these numbers have an average of 80. The average of the other ten numbers is

(A) 60 **(B)** 4 **(C)** 72 **(D)** 40 **(E)** 78

14. Four friends went fishing one day and caught a total of 11 fish.
Each person caught at least one fish.
All of the following statements *could* be true.
Which one of the statements *must* be true?

(A) At least one person caught exactly one fish.
(B) At least one person caught exactly three fish.
(C) At least one person caught more than three fish.
(D) At least one person caught fewer than three fish.
(E) At least two people each caught more than one fish.

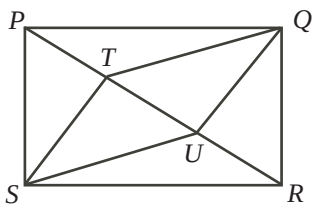
15. The number of positive integers p for which $-1 < \sqrt{p} - \sqrt{100} < 1$ is

(A) 19 **(B)** 21 **(C)** 38 **(D)** 39 **(E)** 41

16. Positive integers a and b satisfy $ab = 2010$. If $a > b$, the smallest possible value of $a - b$ is

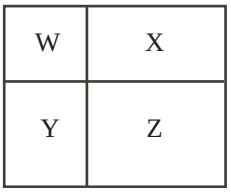
(A) 37 **(B)** 119 **(C)** 191 **(D)** 1 **(E)** 397

17. In the diagram, $PQRS$ is a rectangle with $PQ = 5$ and $QR = 3$. PR is divided into three segments of equal length by points T and U. The area of quadrilateral $STQU$ is



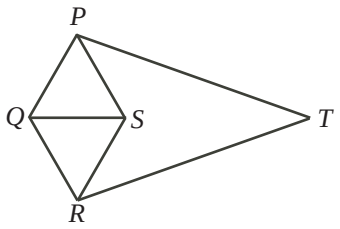
(A) $\frac{17}{3}$ **(B)** 5 **(C)** $\frac{5}{2}$
(D) $\frac{\sqrt{34}}{3}$ **(E)** $\sqrt{34}$

18. A rectangle is divided into four smaller rectangles, labelled W, X, Y, and Z, as shown. The perimeters of rectangles W, X and Y are 2, 3 and 5, respectively. What is the perimeter of rectangle Z?



(A) 6 **(B)** 7 **(C)** 4
(D) 8 **(E)** 7.5

19. In the diagram, $PQ = QR = RS = SP = SQ = 6$ and $PT = RT = 14$. The length of ST is



(A) $4\sqrt{10} - 3$ **(B)** 11 **(C)** $7\sqrt{3} - 3$
(D) 10 **(E)** $\sqrt{232 - 84\sqrt{3}}$

20. A square has side length 5. In how many different locations can point X be placed so that the distances from X to the four sides of the square are 1, 2, 3, and 4?

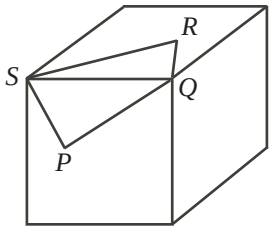
(A) 0 **(B)** 12 **(C)** 4 **(D)** 8 **(E)** 16

Part C: Each correct answer is worth 8.

21. If $\dfrac{x-y}{z-y} = -10$, then the value of $\dfrac{x-z}{y-z}$ is

(A) 11 **(B)** -10 **(C)** 9 **(D)** -9 **(E)** 10

22. A rectangular piece of paper, $PQRS$, has $PQ = 20$ and $QR = 15$. The piece of paper is glued flat on the surface of a large cube so that Q and S are at vertices of the cube. (Note that $\triangle QPS$ and $\triangle QRS$ lie flat on the front and top faces of the cube, respectively.) The shortest distance from P to R, as measured through the cube, is closest to

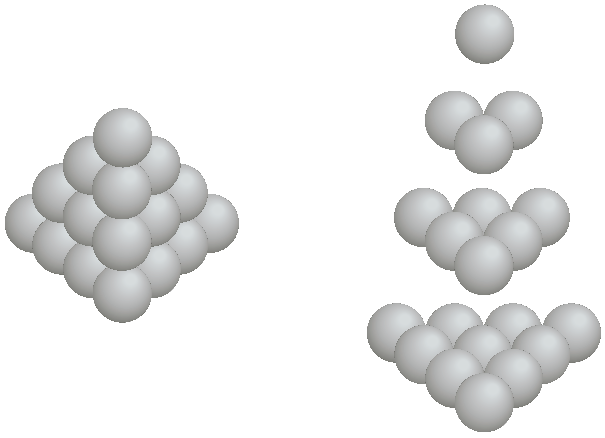


(A) 17.0 **(B)** 25.0 **(C)** 31.0
(D) 17.7 **(E)** 18.4

23. Let t_n equal the integer closest to $\sqrt{n}$.
For example, $t_1 = t_2 = 1$ since $\sqrt{1} = 1$ and $\sqrt{2} \approx 1.41$ and $t_3 = 2$ since $\sqrt{3} \approx 1.73$.
The sum $\dfrac{1}{t_1} + \dfrac{1}{t_2} + \dfrac{1}{t_3} + \dfrac{1}{t_4} + \cdots + \dfrac{1}{t_{2008}} + \dfrac{1}{t_{2009}} + \dfrac{1}{t_{2010}}$ equals

(A) $88\frac{1}{6}$ **(B)** $88\frac{1}{2}$ **(C)** $88\frac{2}{3}$ **(D)** $88\frac{1}{3}$ **(E)** 90

24. Spheres can be stacked to form a tetrahedron by using triangular layers of spheres. Each sphere touches the three spheres below it. The diagrams show a tetrahedron with four layers and the layers of such a tetrahedron. An *internal sphere* in the tetrahedron is a sphere that touches exactly three spheres in the layer above. For example, there is one internal sphere in the fourth layer, but no internal spheres in the first three layers.



A tetrahedron of spheres is formed with thirteen layers and each sphere has a number written on it. The top sphere has a 1 written on it and each of the other spheres has written on it the number equal to the sum of the numbers on the spheres in the layer above with which it is in contact. For the whole thirteen layer tetrahedron, the sum of the numbers on all of the internal spheres is

(A) 772 588 **(B)** 772 566 **(C)** 772 156 **(D)** 772 538 **(E)** 772 626

25. Alex chose positive integers a, b, c, d, e, f and completely multiplied out the polynomial product

$$(1-x)^a(1+x)^b(1-x+x^2)^c(1+x^2)^d(1+x+x^2)^e(1+x+x^2+x^3+x^4)^f$$

After she simplified her result, she discarded any term involving x to any power larger than 6 and was astonished to see that what was left was $1-2x$. If $a > d+e+f$ and $b > c+d$ and $e > c$, what value of a did she choose?

(A) 17 **(B)** 19 **(C)** 20 **(D)** 21 **(E)** 23

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Wednesday, February 18, 2009

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Scoring:	There is *no penalty* for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. The value of $3 + 3^3$ is

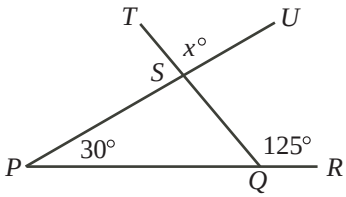
   (A) 12 **(B)** 18 **(C)** 216 **(D)** 30 **(E)** 36

2. If $3 \times 2 + 8 = \nabla + 5$, then ∇ equals

   (A) 14 **(B)** 25 **(C)** 19 **(D)** 17 **(E)** 9

3. In the diagram, PQR, QST and PSU are straight lines. The value of x is

   (A) 75 **(B)** 85 **(C)** 95 **(D)** 125 **(E)** 155



4. If $w = 4$, $x = 9$, and $z = 25$ then $\sqrt{\dfrac{w}{x}} + \sqrt{\dfrac{x}{z}}$ equals

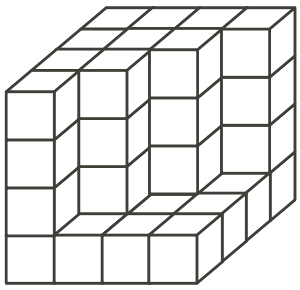
   (A) $\dfrac{5}{8}$ **(B)** $\dfrac{19}{15}$ **(C)** $\dfrac{77}{225}$ **(D)** $\dfrac{181}{225}$ **(E)** $\dfrac{2}{5}$

5. $1 - 4(3-1)^{-1}$ is equal to

   (A) -1 **(B)** $-\frac{3}{2}$ **(C)** 9 **(D)** 6 **(E)** $\frac{11}{3}$

6. Sixty-four identical cubes are stacked in a $4 \times 4 \times 4$ arrangement and then some of the cubes are removed from the front as shown. No cube hidden from sight has been removed. How many cubes remain in the arrangement?

   (A) 46 **(B)** 40 **(C)** 52 **(D)** 55 **(E)** 49



7. If $n > 0$ and $\sqrt{n^2 + n^2 + n^2 + n^2} = 64$, then n equals

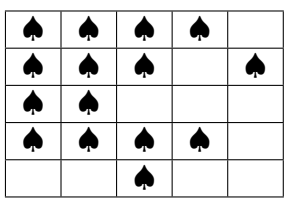
   (A) $\sqrt{8}$ **(B)** 16 **(C)** 4 **(D)** 32 **(E)** $\sqrt{2}$

8. Gavin has a collection of 50 songs that are each 3 minutes in length and 50 songs that are each 5 minutes in length. What is the maximum number of songs from his collection that he can play in 3 hours?

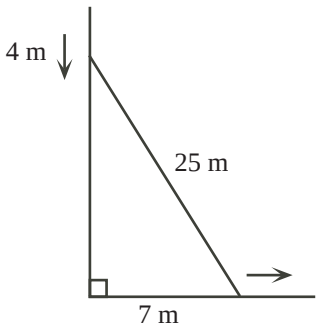
   (A) 100 **(B)** 36 **(C)** 56 **(D)** 60 **(E)** 45

9. In the diagram, any ♠ may be moved to any unoccupied space. What is the smallest number of ♠'s that must be moved so that each row and each column contains three ♠'s?

   (A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5



10. Judi leans a 25 m ladder against a vertical wall with the bottom of the ladder 7 m from the wall. (Please note that Judi is very strong – don't try this at home!) As she pulls the bottom of the ladder away from the wall, the top of the ladder slides 4 m down the wall. How far did she pull the bottom of the ladder from its original position?



(A) 4 m **(B)** 11 m **(C)** 2 m
(D) 13 m **(E)** 8 m

Part B: Each correct answer is worth 6.

11. Suppose m and n are positive integers with $m < n$. The value of $\dfrac{m+3}{n+3}$ will be

(A) equal to 1
(B) equal to 3
(C) less than the value of $\dfrac{m}{n}$
(D) greater than the value of $\dfrac{m}{n}$
(E) equal to the value of $\dfrac{m}{n}$

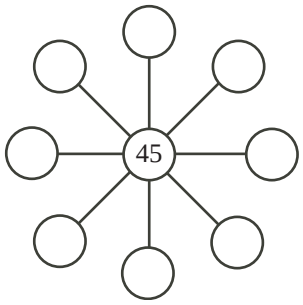
12. How many four-digit integers between 5000 and 6000 are there for which the thousands digit equals the sum of the other three digits? (The thousands digit of 5124 is 5.)

(A) 5 **(B)** 15 **(C)** 21 **(D)** 30 **(E)** 12

13. The number of integers x for which the value of $\dfrac{-6}{x+1}$ is an integer is

(A) 8 **(B)** 9 **(C)** 2 **(D)** 6 **(E)** 7

14. Different positive integers can be written in the eight empty circles so that the product of any three integers in a straight line is 3240. What is the largest possible sum of the eight numbers surrounding 45?

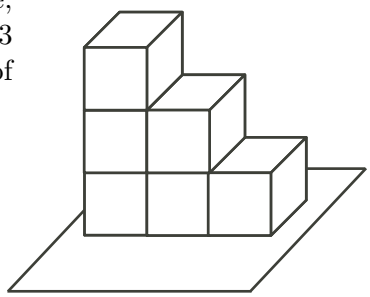


(A) 139 **(B)** 211 **(C)** 156
(D) 159 **(E)** 160

15. On Monday, 10% of the students at Dunkley S.S. were absent and 90% were present. On Tuesday, 10% of those who were absent on Monday were present and the rest of those absent on Monday were still absent. Also, 10% of those who were present on Monday were absent and the rest of those present on Monday were still present. What percentage of the students at Dunkley S.S. were present on Tuesday?

(A) 81% **(B)** 82% **(C)** 90% **(D)** 91% **(E)** 99%

16. Six dice are stacked on the floor as shown. On each die, the 1 is opposite the 6, the 2 is opposite the 5, and the 3 is opposite the 4. What is the maximum possible sum of numbers on the 21 visible faces?



(A) 69 **(B)** 88 **(C)** 89
(D) 91 **(E)** 96

17. In the diagram, the perimeter of the semicircular region is 20. (The perimeter includes both the semicircular arc and the diameter.) The area of the region is closest to

(A) 36.6 **(B)** 23.8 **(C)** 49.3
(D) 51.6 **(E)** 26.7

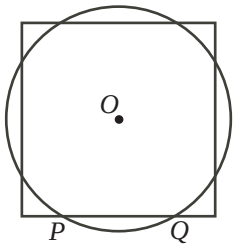
18. On Monday, Hank drove to work at an average speed of 70 km/h and arrived 1 minute late. On Tuesday, he left at the same time and took the same route. This time he drove at an average speed of 75 km/h and arrived 1 minute early. How long is his route to work?

(A) 30 km **(B)** 35 km **(C)** 45 km **(D)** 50 km **(E)** 60 km

19. If $2^x = 15$ and $15^y = 32$, the value of xy is

(A) 5 **(B)** 8 **(C)** 16 **(D)** 6 **(E)** 4

20. In the diagram, the circle and the square have the same centre O and equal areas. The circle has radius 1 and intersects one side of the square at P and Q. What is the length of PQ?



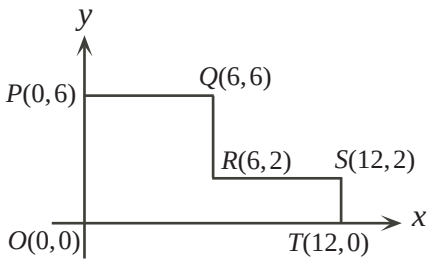
(A) $\sqrt{4-\pi}$ **(B)** 1 **(C)** $\sqrt{2}$
(D) $2-\sqrt{\pi}$ **(E)** $4-\sqrt{\pi}$

Part C: Each correct answer is worth 8.

21. At Matilda's birthday party, the ratio of people who ate ice cream to people who ate cake was 3 : 2. People who ate both ice cream and cake were included in both categories. If 120 people were at the party, what is the maximum number of people who could have eaten both ice cream and cake?

(A) 24 **(B)** 30 **(C)** 48 **(D)** 80 **(E)** 72

22. In the diagram, two straight lines are to be drawn through $O(0,0)$ so that the lines divide the figure $OPQRST$ into 3 pieces of equal area. The sum of the slopes of the lines will be



(A) $\frac{35}{24}$ **(B)** $\frac{7}{6}$ **(C)** $\frac{5}{4}$
(D) $\frac{4}{3}$ **(E)** $\frac{11}{8}$

23. Suppose that a, b, c, and d are positive integers that satisfy the equations

$$\begin{aligned} ab + cd &= 38 \\ ac + bd &= 34 \\ ad + bc &= 43 \end{aligned}$$

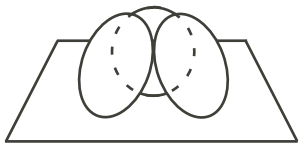
What is the value of $a + b + c + d$?

(A) 15 **(B)** 16 **(C)** 17 **(D)** 18 **(E)** 19

24. Starting with the input (m, n), Machine A gives the output (n, m).
Starting with the input (m, n), Machine B gives the output $(m + 3n, n)$.
Starting with the input (m, n), Machine C gives the output $(m - 2n, n)$.
Natalie starts with the pair $(0, 1)$ and inputs it into one of the machines. She takes the output and inputs it into any one of the machines. She continues to take the output that she receives and inputs it into any one of the machines. (For example, starting with $(0, 1)$, she could use machines B, B, A, C, B in that order to obtain the output $(7, 6)$.) Which of the following pairs is impossible for her to obtain after repeating this process any number of times?

(A) (2009,1016) **(B)** (2009,1004) **(C)** (2009,1002)
(D) (2009,1008) **(E)** (2009,1032)

25. In the diagram, three circles of radius 10 are tangent to each other and to a plane in three-dimensional space. Each of the circles is inclined at 45° to the plane. There are three points where the circles touch each other. These three points lie on a circle parallel to the plane. The radius of this circle is closest to



(A) 6.9 **(B)** 7.1 **(C)** 7.3
(D) 7.5 **(E)** 7.7

Canadian Mathematics Competition

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Canadian Mathematics Competition

An activity of the Centre for Education in Mathematics and Computing, University of Waterloo, Waterloo, Ontario

Fermat Contest (Grade 11)

Tuesday, February 19, 2008

Time: 60 minutes

Calculators are permitted

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Part A: Each correct answer is worth 5.

1. The value of $\dfrac{1^2+2^2+3^2+4^2}{1\times 2\times 3}$ is

   (A) 110 **(B)** 22 **(C)** $\dfrac{50}{3}$ **(D)** 5 **(E)** 14

2. The value of $6\left(\frac{3}{2}+\frac{2}{3}\right)$ is

   (A) 13 **(B)** 6 **(C)** $\frac{13}{6}$ **(D)** $\frac{29}{3}$ **(E)** 5

3. If $1+2+3+4+5+x=21+22+23+24+25$, then the value of x is

   (A) 11 **(B)** 210 **(C)** 100 **(D)** 20 **(E)** 26

4. An empty truck weighs 9600 kg. When the truck is loaded with 40 identical crates, the total weight is 38 000 kg. The weight of each crate is

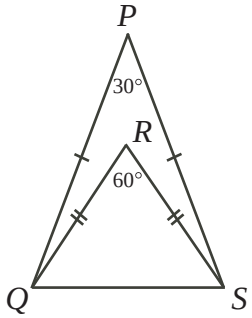
   (A) 460 kg **(B)** 950 kg **(C)** 1190 kg **(D)** 240 kg **(E)** 710 kg

5. If $\dfrac{18}{\sqrt{x}}=2$, then the value of x is

   (A) 81 **(B)** 36 **(C)** 18 **(D)** 9 **(E)** 3

6. In the diagram, what is the measure of $\angle PQR$?

   (A) 45° **(B)** 30° **(C)** 60°
   (D) 75° **(E)** 15°



7. If p is an odd integer and q is an even integer, which one of the following is an odd integer?

   (A) $2p+3q$ **(B)** $3p+2q$ **(C)** $4p+q$ **(D)** $2(p+3q)$ **(E)** pq

8. Two 3-digit integers, abc and def, have the following property:

$$\begin{array}{cccc} & a & b & c \\ + & d & e & f \\ \hline 1 & 0 & 0 & 0 \end{array}$$

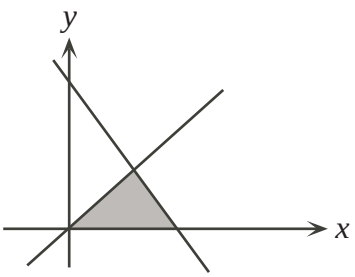
   None of a, b, c, d, e, or f is 0. What is $a+b+c+d+e+f$?

   (A) 10 **(B)** 19 **(C)** 21 **(D)** 28 **(E)** 30

9. Beshmi invested $\frac{1}{5}$ of her savings in Company X, 42% in Company Y, and the remainder in Company Z. If Beshmi invested \$10 500 in Company Y, how much did she invest in Company Z?

(A) \$25 000 **(B)** \$15 500 **(C)** \$14 000 **(D)** \$9500 **(E)** \$5000

10. In the diagram, the shaded region is bounded by the x-axis and the lines $y = x$, and $y = -2x + 3$. The area of the shaded region is



(A) $\frac{3}{4}$ **(B)** $\frac{3}{2}$ **(C)** $\frac{9}{4}$

(D) 1 **(E)** $\frac{\sqrt{10}}{4}$

Part B: Each correct answer is worth 6.

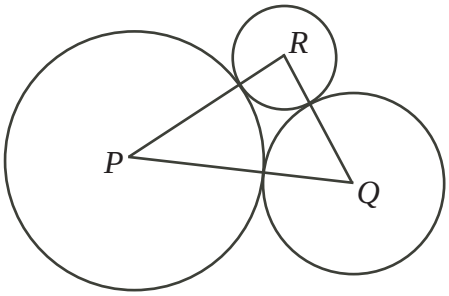
11. If $\dfrac{1}{x} = 2$ and $\dfrac{1}{x} + \dfrac{3}{y} = 3$, then the value of $x + y$ is

(A) 3 **(B)** $\frac{5}{6}$ **(C)** $\frac{7}{3}$ **(D)** $\frac{7}{2}$ **(E)** $\frac{4}{3}$

12. On seven tests, each out of 100 marks, Siobhan received marks of 69, 53, 69, 71, 78, x, and y. If her average mark on the seven tests is 66, then the minimum possible value of x is

(A) 22 **(B)** 68 **(C)** 61 **(D)** 53 **(E)** 0

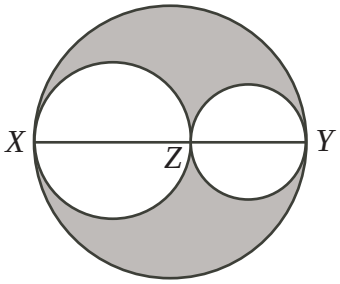
13. In the diagram, the circles with centres P, Q and R have radii 3, 2 and 1 respectively. Each circle touches the other two as shown. The area of $\triangle PQR$ is



(A) 12 **(B)** 6 **(C)** 7.5

(D) 10 **(E)** 4

14. In the diagram, Z lies on XY and the three circles have diameters XZ, ZY and XY. If $XZ = 12$ and $ZY = 8$, then the ratio of the area of the shaded region to the area of the unshaded region is



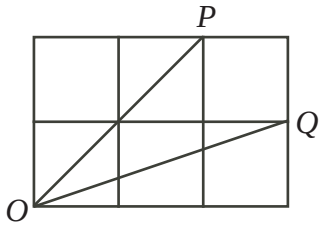
(A) 12 : 25 **(B)** 12 : 13 **(C)** 1 : 1

(D) 1 : 2 **(E)** 2 : 3

15. In a relay race, Ainslee runs the first lap in 72 seconds. Bridget runs the next lap at $\frac{9}{10}$ of Ainslee's speed. Cecilia runs the next lap at $\frac{4}{3}$ of Bridget's speed. Dana runs the last lap at $\frac{6}{5}$ of Cecilia's speed. What is their total time, to the nearest second?

(A) 4 minutes, 48 seconds
(B) 4 minutes, 22 seconds
(C) 5 minutes, 27 seconds
(D) 4 minutes, 37 seconds
(E) 3 minutes, 46 seconds

16. In the diagram, the six small squares all have side length 2. Lines are drawn from O to P and O to Q. The measure of $\angle POQ$ in degrees, accurate to one decimal place, is



(A) 15.0 **(B)** 25.5 **(C)** 26.6
(D) 22.5 **(E)** 30.0

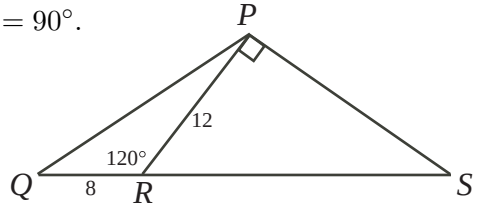
17. The difference between the squares of two consecutive integers is 199. The sum of the squares of these two consecutive integers is

(A) 19 801 **(B)** 39 601 **(C)** 19 602 **(D)** 20 201 **(E)** 19 405

18. An *arithmetic sequence* is a sequence in which each term after the first is obtained by adding a constant to the previous term.
If the first four terms of an arithmetic sequence are a, $2a$, b, and $a - 6 - b$ for some numbers a and b, then the value of the 100th term is

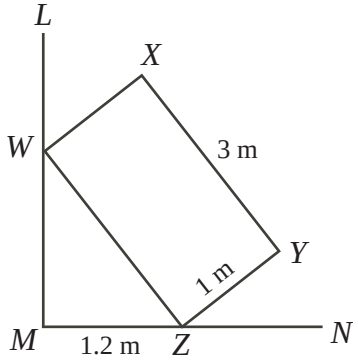
(A) -100 **(B)** -300 **(C)** 150 **(D)** -150 **(E)** 100

19. In the diagram, R is on QS and $QR = 8$.
Also, $PR = 12$, $\angle PRQ = 120°$, and $\angle RPS = 90°$.
What is the area of $\triangle QPS$?



(A) $72\sqrt{3}$ **(B)** 72 **(C)** 36
(D) $60\sqrt{3}$ **(E)** $96\sqrt{3}$

20. In the diagram, LM is perpendicular to MN. Rectangle $WXYZ$ has W on LM and Z on MN. Also, $YZ = 1$ m, $XY = 3$ m and $MZ = 1.2$ m. What is the distance from X to line MN, to the nearest hundredth of a metre?



(A) 2.75 m **(B)** 3.67 m **(C)** 3.15 m
(D) 3.26 m **(E)** 3.63 m

Part C: Each correct answer is worth 8.

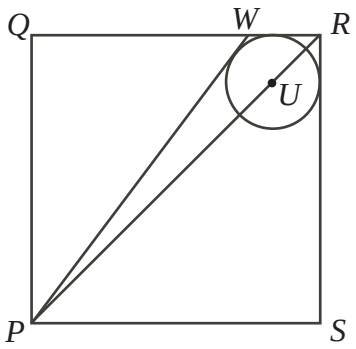
21. Suppose $N = 1 + 11 + 101 + 1001 + 10001 + \ldots + 1\overbrace{000\ldots00}^{50 \text{ zeroes}}1$.
When N is calculated and written as a single integer, the sum of its digits is

(A) 58 **(B)** 99 **(C)** 55 **(D)** 50 **(E)** 103

22. For how many integers k do the parabolas with equations $y = -\frac{1}{8}x^2 + 4$ and $y = x^2 - k$ intersect on or above the x-axis?

(A) 9 **(B)** 32 **(C)** 33 **(D)** 36 **(E)** 37

23. Square $PQRS$ has side length 4 m. Point U is on PR with $PR = 4UR$. A circle centered at U touches two sides of the square. PW is a tangent to the circle, with W on QR. The length of PW, to the nearest thousandth of a metre, is

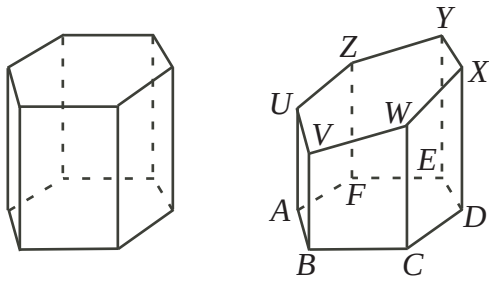


(A) 4.123 m **(B)** 4.472 m **(C)** 4.685 m
(D) 4.726 m **(E)** 4.767 m

24. The number of triples (a, b, c) of positive integers such that $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{3}{4}$ is

(A) 16 **(B)** 25 **(C)** 31 **(D)** 19 **(E)** 34

25. A right regular hexagonal prism is sliced as shown in the diagram. The bottom of the new solid is a regular hexagon $ABCDEF$. The six side faces are trapezoids perpendicular to $ABCDEF$. The top is a hexagon $UVWXYZ$ that is not necessarily a regular hexagon.



Of the six edges AU, BV, CW, DX, EY, and FZ, three have lengths 4, 7 and 10. The largest possible value for $AU + BV + CW + DX + EY + FZ$ is

(A) 42 **(B)** 51 **(C)** 69 **(D)** 78 **(E)** 91

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8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor tells you to begin, you will have *sixty* minutes of working time.

The names of some top-scoring students will be published in the PCF Results on our Web site, http://www.cemc.uwaterloo.ca.

Scoring:	There is *no penalty* for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. The value of $\dfrac{36-12}{12-4}$ is

   (A) 6 **(B)** 9 **(C)** 1 **(D)** 31 **(E)** 3

2. If $7x = 28$ and $x + w = 9$, what is the value of xw?

   (A) 9 **(B)** 20 **(C)** 18 **(D)** 52 **(E)** -252

3. Of the fractions $\dfrac{3}{4}$, $\dfrac{7}{8}$, $\dfrac{13}{16}$, and $\dfrac{1}{2}$, what is the difference between the largest and the smallest?

   (A) $\dfrac{3}{8}$ **(B)** $\dfrac{6}{7}$ **(C)** $\dfrac{5}{16}$ **(D)** $\dfrac{1}{16}$ **(E)** $\dfrac{1}{8}$

4. When $x = -5$, the value of $-2x^2 + \dfrac{5}{x}$ is

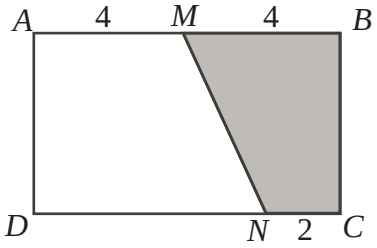
   (A) 99 **(B)** 101 **(C)** -51 **(D)** 19 **(E)** -49

5. What is the value of $1^{-2} + 2^{-1}$?

   (A) $\frac{3}{2}$ **(B)** $\frac{1}{27}$ **(C)** 4 **(D)** -4 **(E)** 9

6. In the diagram, the area of rectangle $ABCD$ is 40. The area of $MBCN$ is

   (A) 15 **(B)** 10 **(C)** 30 **(D)** 12 **(E)** 16



7. The product of three positive integers is 42. The sum of two of these integers is 9. The third integer is

   (A) 1 **(B)** 7 **(C)** 6 **(D)** 3 **(E)** 2

8. Ivan trained for a cross-country meet.
   On Monday, he ran a certain distance.
   On Tuesday, he ran twice as far as he ran on Monday.
   On Wednesday, he ran half as far as he ran on Tuesday.
   On Thursday, he ran half as far as he ran on Wednesday.
   On Friday, he ran twice as far as he ran on Thursday.
   If the shortest distance that he ran on any of the five days is 5 km, how far did he run in total?

   (A) 55 km **(B)** 25 km **(C)** 27.5 km **(D)** 17.5 km **(E)** 50 km

9. If $\dfrac{1}{x+3} = 2$, then the value of $\dfrac{1}{x+5}$ is

   (A) $\frac{1}{2}$ **(B)** $\frac{2}{3}$ **(C)** $\frac{2}{5}$ **(D)** $\frac{1}{4}$ **(E)** 4

10. A store normally sells each of its DVDs for \$20. At a sale, Phyllis buys two DVDs at the regular price and gets a third DVD for half price. This is the same rate of discount as getting

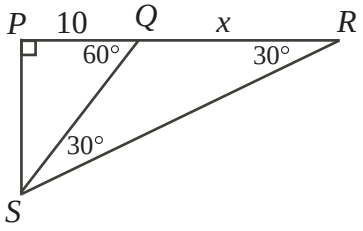
(A) 2 for the price of 1
(B) 3 for the price of 2
(C) 4 for the price of 3
(D) 5 for the price of 4
(E) 6 for the price of 5

Part B: Each correct answer is worth 6.

11. Five numbers in increasing order are 2, 5, x, 10, and y. The median of the numbers is 7 and the mean (average) is 8. The value of y is

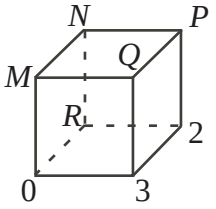
(A) 16 **(B)** 14 **(C)** 15 **(D)** 18 **(E)** 12

12. In the diagram, $PQ = 10$ and $QR = x$. The value of x is



(A) $10\sqrt{3}$ **(B)** 20 **(C)** $\frac{50}{3}$
(D) $\frac{20}{\sqrt{3}}$ **(E)** 10

13. In the diagram, each of the numbers 0, 1, 2, 3, 4, 5, 6, and 7 is to be used to label a vertex of the cube. The numbers 0, 2 and 3 are placed as shown. The sum of the numbers at the ends of each edge must be a prime number. (Note: 1 is not a prime number.) The value of $M + N + P + Q$ must be



(A) 16 **(B)** 17 **(C)** 18
(D) 19 **(E)** 22

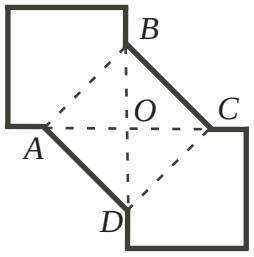
14. Two positive integers a and b have the property that if a is increased by 25%, the result will be *greater than* five times the value of b. What is the minimum possible value for $a + b$?

(A) 3 **(B)** 6 **(C)** 10 **(D)** 9 **(E)** 21

15. How many three-digit positive integers x are there with the property that x and $2x$ have only even digits? (One such number is $x = 420$, since $2x = 840$ and each of x and $2x$ has only even digits.)

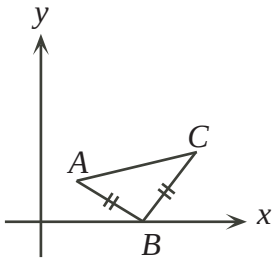
(A) 64 **(B)** 18 **(C)** 16 **(D)** 125 **(E)** 100

16. In the diagram, each of the three squares has a side length of 3. Two of the squares have a common vertex O, and O is the centre of the square labelled $ABCD$. The perimeter of the entire figure is closest to



(A) 21.5 **(B)** 22.0 **(C)** 22.5
(D) 24.0 **(E)** 30.0

17. In the diagram, $A(2,2)$ and $C(8,4)$ are two of the vertices of an isosceles right-angled triangle ABC. If the vertex B is located on the x-axis and $\angle ABC = 90°$, the x-coordinate of B is

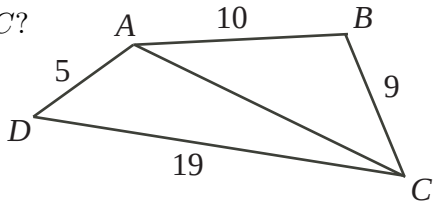


(A) 3 **(B)** 4 **(C)** 5
(D) 6 **(E)** 7

18. Alphonso and Karen started out with the *same number* of apples. Karen gave twelve of her apples to Alphonso. Next, Karen gave half of her remaining apples to Alphonso. If Alphonso now has four times as many apples as Karen, how many apples does Karen now have?

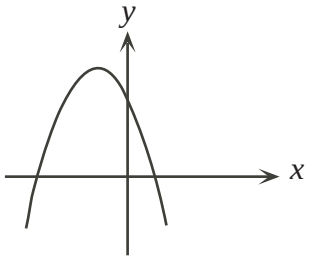
(A) 12 **(B)** 24 **(C)** 36 **(D)** 48 **(E)** 72

19. In the diagram, $ABCD$ is a quadrilateral with diagonal AC. Which of the following is a possible length for AC?



(A) 9 **(B)** 10 **(C)** 13
(D) 15 **(E)** 20

20. The graph of the function $y = ax^2 + bx + c$ is shown in the diagram. Which of the following must be positive?



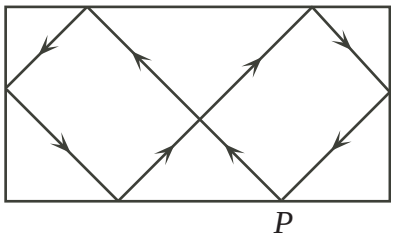
(A) a **(B)** bc **(C)** ab^2
(D) $b - c$ **(E)** $c - a$

Part C: Each correct answer is worth 8.

21. Five consecutive positive integers have the property that the sum of the second, third and fourth is a perfect square, while the sum of all five is a perfect cube. If m is the third of these five integers, then the minimum possible value of m satisfies

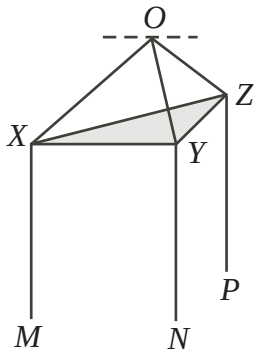
(A) $m \leq 200$
(B) $200 < m \leq 400$
(C) $400 < m \leq 600$
(D) $600 < m \leq 800$
(E) $m > 800$

22. A ball placed at point P on a rectangular billiard table is shot at an angle of 45° to the edge of the table. After successively bouncing off the edges of the table at 45° angles, it returns to point P, as shown. If the ball travels 7 m, the perimeter, in metres, of the table is closest to



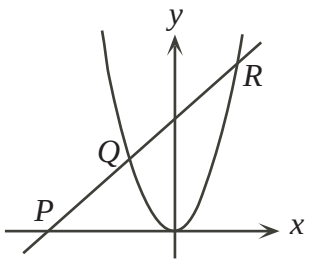
(A) 7.0 **(B)** 7.5 **(C)** 8.0
(D) 8.5 **(E)** 9.0

23. An ugly light fixture is hanging from point O on the ceiling. Wires OXM, OYN and OZP pass through the vertices of a very thin wooden equilateral triangle XYZ of side 60 cm. (A small bulb is attached to the end of each wire.) The plane of the wooden triangle is parallel to the ceiling. If each wire is 100 cm long and the lower end of each wire is 90 cm from the ceiling, what is the vertical distance between the wooden triangle and the ceiling?



(A) 40 cm **(B)** 45 cm **(C)** 50 cm
(D) 55 cm **(E)** 60 cm

24. A line with slope 1 passes through point P on the negative x-axis and intersects the parabola $y = x^2$ at points Q and R, as shown. If $PQ = QR$, then the y-intercept of PR is closest to



(A) 9.9 **(B)** 10.2 **(C)** 8.2
(D) 9.3 **(E)** 8.6

25. How many ordered pairs (b, g) of positive integers with $4 \leq b \leq g \leq 2007$ are there such that when b black balls and g gold balls are randomly arranged in a row, the probability that the balls on each end have the same colour is $\frac{1}{2}$?

(A) 60 **(B)** 62 **(C)** 58 **(D)** 61 **(E)** 59

Canadian Mathematics Competition

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Thank you for writing the 2007 Fermat Contest!
In 2006, more than 90 000 students around the world registered to write the Pascal, Cayley and Fermat Contests.

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- **Learn about workshops and resources we offer for teachers**
- **Find your school results**

Canadian Mathematics Competition

An activity of the Centre for Education in Mathematics and Computing, University of Waterloo, Waterloo, Ontario

Fermat Contest (Grade 11)

Wednesday, February 22, 2006

Time: 60 minutes

Calculators are permitted

Instructions

1. Do not open the Contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper left corner.
5. **Be certain that you code your name, age, sex, grade, and the Contest you are writing in the response form. Only those who do so can be counted as official contestants.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
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9. When your supervisor tells you to begin, you will have *sixty* minutes of working time.

Scoring:	There is *no penalty* for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. The value of $\dfrac{1}{4 \times 5}$ is

   (A) 0.45 **(B)** 0.05 **(C)** 1.25 **(D)** 0.20 **(E)** 0.02

2. If $2x + 3x + 4x = 12 + 9 + 6$, then x equals

   (A) 6 **(B)** 3 **(C)** 1 **(D)** $\frac{1}{3}$ **(E)** $10\frac{1}{2}$

3. The value of $\dfrac{4^3}{10^2 - 6^2}$ is

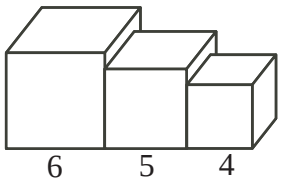
   (A) 1 **(B)** 0.5 **(C)** -35.36 **(D)** 1.5 **(E)** 4

4. The value of $\left(\sqrt{\sqrt{9} + \sqrt{1}}\right)^4$ is

   (A) $\sqrt{10}$ **(B)** 10 **(C)** 16 **(D)** 82 **(E)** 100

5. Three cubes have edges of lengths 4, 5 and 6.
   The average (mean) of their volumes is

   (A) 120 **(B)** 125 **(C)** 1125
   (D) 261 **(E)** 135



6. The regular price for a T-shirt is \$25 and the regular price for a pair of jeans is \$75. If the T-shirt is sold at a 30% discount and the jeans are sold at a 10% discount, then the total discount is

   (A) \$15 **(B)** \$20 **(C)** \$30 **(D)** \$36 **(E)** \$40

7. What is the smallest positive integer p for which $\sqrt{2^3 \times 5 \times p}$ is an integer?

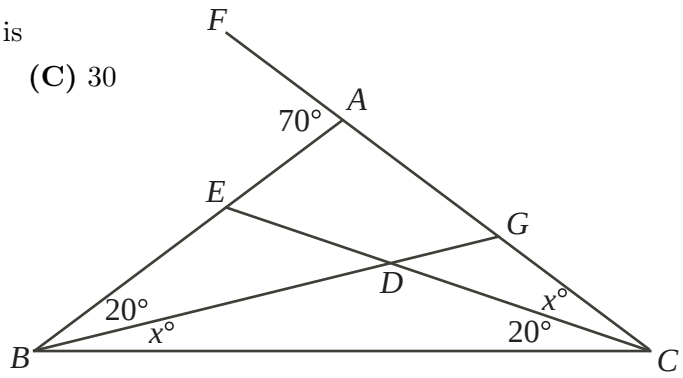
   (A) 2 **(B)** 5 **(C)** 10 **(D)** 1 **(E)** 20

8. If Corina had added the numbers P and Q correctly, the answer would have been 16. By mistake, she subtracted Q from P. Her answer was 4. What is the value of P?

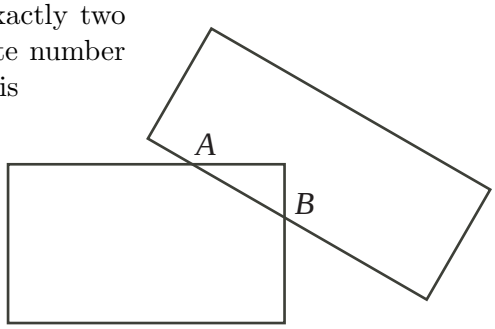
   (A) 4 **(B)** 5 **(C)** 8 **(D)** 10 **(E)** 16

9. In the diagram, the value of x is

   (A) 15 **(B)** 20 **(C)** 30
   (D) 35 **(E)** 50



10. In the diagram, two rectangles intersect at exactly two points, A and B. The maximum possible finite number of points of intersection of *any* two rectangles is



(A) 3 **(B)** 4 **(C)** 12 **(D)** 8 **(E)** 6

Part B: Each correct answer is worth 6.

11. If $\frac{a}{b} = 3$ and $\frac{b}{c} = 2$, then the value of $\frac{a-b}{c-b}$ is

(A) -4 **(B)** $-\frac{1}{3}$ **(C)** $\frac{2}{3}$ **(D)** 2 **(E)** 6

12. If $\left(2^4\right)\left(3^6\right) = 9\left(6^x\right)$, what is the value of x?

(A) 2 **(B)** 3 **(C)** 4 **(D)** 216 **(E)** 8

13. In 2004, Gerry downloaded 200 songs. In 2005, Gerry downloaded 360 songs at a cost per song which was 32 cents less than in 2004. Gerry's *total* cost each year was the same. The cost of downloading the 360 songs in 2005 was

(A) \$144.00 **(B)** \$108.00 **(C)** \$80.00 **(D)** \$259.20 **(E)** \$72.00

14. If the system of equations

$$\begin{aligned} px + qy &= 8 \\ 3x - qy &= 38 \end{aligned}$$

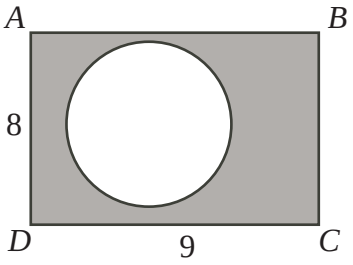
has the solution $(x, y) = (2, -4)$, then p is equal to

(A) -12 **(B)** 20 **(C)** 8 **(D)** 40 **(E)** 21.5

15. The points $(5, 3)$ and $(1, -1)$ are plotted on a sheet of graph paper. The sheet of graph paper is folded along a line so that the point $(5, 3)$ lands on top of the point $(1, -1)$. The equation of the line that represents the fold is

(A) $y = -x + 1$ **(B)** $y = -x + 2$ **(C)** $y = -x + 3$
(D) $y = -x + 4$ **(E)** $y = -x + 5$

16. In the diagram, $ABCD$ is a rectangle. If the area of the circle is equal to the area of the shaded region, the radius of the circle is



(A) $\sqrt{\frac{6}{\pi}}$ **(B)** $\frac{6}{\pi}$ **(C)** $\frac{6}{\sqrt{\pi}}$
(D) $\sqrt{\frac{18}{\pi}}$ **(E)** $\frac{18}{\pi}$

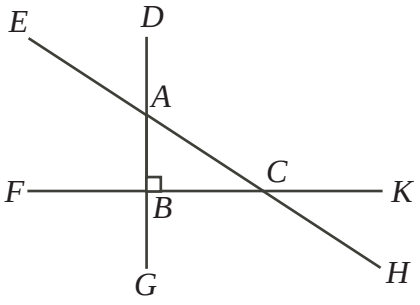
17. In seven term sequence, 5, p, q, 13, r, 40, x, each term after the third term is the sum of the preceding three terms. The value of x is

(A) 21 **(B)** 61 **(C)** 67 **(D)** 74 **(E)** 80

18. The front wheel of Georgina's bicycle has a diameter of 0.75 metres. She cycled for 6 minutes at a speed of 24 kilometres per hour. The number of complete rotations that the wheel made during this time is closest to

(A) 610 **(B)** 1020 **(C)** 1360 **(D)** 1700 **(E)** 5430

19. In the diagram, $\triangle ABC$ is right-angled. Side AB is extended in each direction to points D and G such that $DA = AB = BG$. Similarly, BC is extended to points F and K so that $FB = BC = CK$, and AC is extended to points E and H so that $EA = AC = CH$. The ratio of the area of the hexagon $DEFGHK$ to the area of $\triangle ABC$ is



(A) 4 : 1 **(B)** 7 : 1 **(C)** 9 : 1

(D) 16 : 1 **(E)** 13 : 1

20. A bag contains eight yellow marbles, seven red marbles, and five black marbles. Without looking in the bag, Igor removes N marbles all at once. If he is to be sure that, no matter which choice of N marbles he removes, there are at least four marbles of one colour and at least three marbles of another colour left in the bag, what is the maximum possible value of N?

(A) 6 **(B)** 7 **(C)** 8 **(D)** 9 **(E)** 10

Part C: Each correct answer is worth 8.

21. For how many integers n, with $2 \leq n \leq 80$, is $\dfrac{(n-1)(n)(n+1)}{8}$ equal to an integer?

(A) 10 **(B)** 20 **(C)** 59 **(D)** 39 **(E)** 49

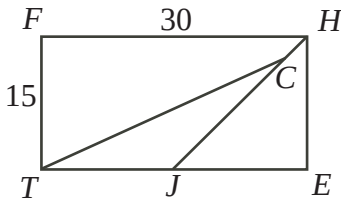
22. Quincy and Celine have to move 16 small boxes and 10 large boxes. The chart indicates the time that each person takes to move each type of box. They start moving the boxes at 9:00 a.m. The earliest time at which they can be finished moving all of the boxes is

	Celine	Quincy
small box	2 min.	3 min.
large box	6 min.	5 min.

(A) 9:41 a.m. **(B)** 9:42 a.m. **(C)** 9:43 a.m.

(D) 9:44 a.m. **(E)** 9:45 a.m.

23. Rectangle $TEHF$ has dimensions 15 m by 30 m, as shown. Tom the Cat begins at T, and Jerry the Mouse begins at J, the midpoint of TE. Jerry runs at 3 m/s in a straight line towards H. Tom starts at the same time as Jerry, and, running at 5 m/s in a straight line, arrives at point C at the same time as Jerry. The time, in seconds, that it takes Tom to catch Jerry is closest to



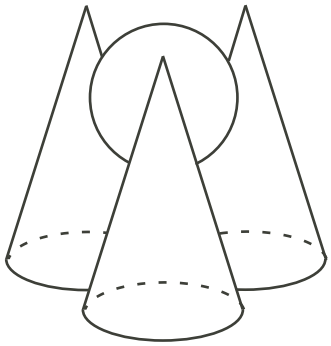
(A) 5.4 **(B)** 5.6 **(C)** 5.8

(D) 6.0 **(E)** 6.2

24. If a and b are positive integers such that $\frac{1}{a} + \frac{1}{2a} + \frac{1}{3a} = \frac{1}{b^2 - 2b}$, then the smallest possible value of $a + b$ is

(A) 8 **(B)** 6 **(C)** 96 **(D)** 10 **(E)** 50

25. Three identical cones each have a radius of 50 and a height of 120. The cones are placed so that their circular bases are touching each other. A sphere is placed so that it rests in the space created by the three cones, as shown. If the top of the sphere is level with the tops of the cones, then the radius of the sphere is closest to



(A) 38.9 **(B)** 38.7 **(C)** 38.1

(D) 38.5 **(E)** 38.3

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Canadian Mathematics Competition

An activity of the Centre for Education in Mathematics and Computing, University of Waterloo, Waterloo, Ontario

Fermat Contest (Grade 11)

Wednesday, February 23, 2005

Time: 60 minutes

Calculators are permitted.

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Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. The value of $\dfrac{150 + (150 \div 10)}{15 - 5}$ is

   (A) 6 **(B)** 3 **(C)** 146 **(D)** 151.5 **(E)** 16.5

2. $\frac{1}{2} - \frac{1}{3} + \frac{3}{9}$ equals

   (A) $\frac{1}{4}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{5}{18}$ **(D)** $\frac{1}{9}$ **(E)** 0

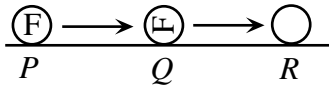
3. If $a = \frac{1}{2}$ and $b = \frac{2}{3}$, then $\dfrac{6a + 18b}{12a + 6b}$ equals

   (A) 9 **(B)** 7 **(C)** 10 **(D)** 6 **(E)** $\frac{3}{2}$

4. If $\sqrt{4 + 9 + x^2} = 7$, then a possible value for x is

   (A) 6 **(B)** 2 **(C)** 4 **(D)** 36 **(E)** 0

5. A Fermat coin rolls from P to Q to R, as shown. If the distance from P to Q is equal to the distance from Q to R, what is the orientation of the coin when it reaches R?

   (A)  **(B)**  **(C)** 

   (D)  **(E)** 

6. The sum of the first 2005 terms of the sequence 1, 2, 3, 4, 1, 2, 3, 4, . . . is

   (A) 5011 **(B)** 5110 **(C)** 5020 **(D)** 5010 **(E)** 501

7. In triangle ABC, $\angle A$ is 21° more than $\angle B$, and $\angle C$ is 36° more than $\angle B$. The size of $\angle B$ is

   (A) 20° **(B)** 41° **(C)** 62° **(D)** 46° **(E)** 56°

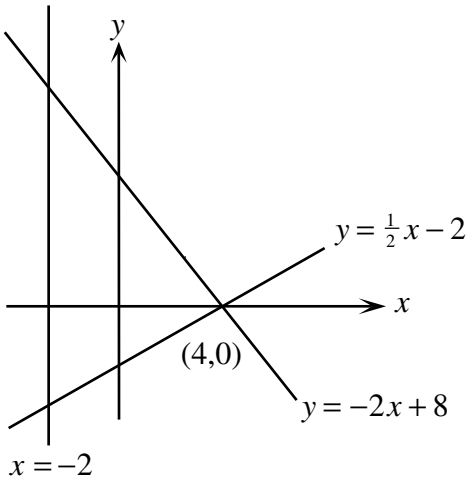
8. Seven children, each with the same birthday, were born in seven consecutive years. The sum of the ages of the youngest three children is 42. What is the sum of the ages of the oldest three?

   (A) 51 **(B)** 54 **(C)** 57 **(D)** 60 **(E)** 63

9. The lines $y = -2x + 8$ and $y = \frac{1}{2}x - 2$ meet at $(4, 0)$, as shown. The area of the triangle formed by these two lines and the line $x = -2$ is

(A) 15 **(B)** 27 **(C)** 30

(D) 36 **(E)** 45



10. If 50% of P equals 20% of Q, then P, as a percent of Q, is

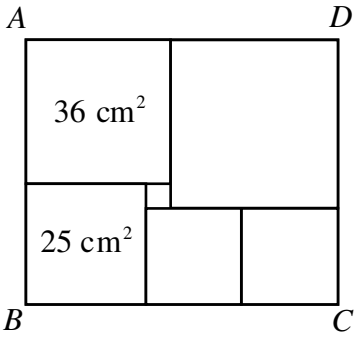
(A) 60% **(B)** 250% **(C)** 40% **(D)** 20% **(E)** 30%

Part B: Each correct answer is worth 6.

11. Rectangle $ABCD$ is made up of six squares. The areas of two of the squares are shown. The perimeter of rectangle $ABCD$, in centimetres, is

(A) 50 **(B)** 44 **(C)** 46

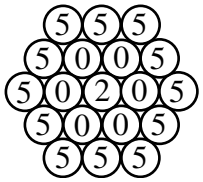
(D) 52 **(E)** 48



12. Starting with the 2 in the centre, the number 2005 can be formed by moving from circle to circle only if the two circles are touching. How many different paths can be followed to form 2005?

(A) 36 **(B)** 24 **(C)** 12

(D) 18 **(E)** 6



13. A circle is drawn so that no part of it lies outside a *regular* hexagon. If such a circle does not touch all six sides of the hexagon, what is the maximum number of sides that it could touch?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

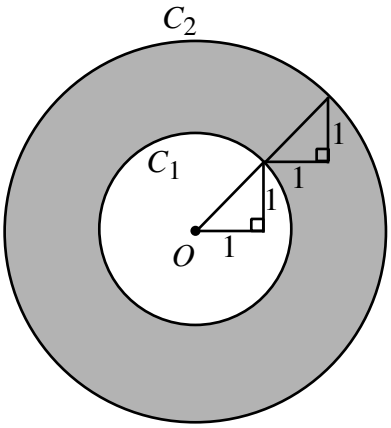
14. The weight of a lioness is six times the weight of her female cub and four times the weight of her male cub. If the difference between the weights of the male and female cub is 14 kg, the weight of the lioness, in kg, is

(A) 84 **(B)** 252 **(C)** 168 **(D)** 140 **(E)** 112

15. If $(x - 4)(5x + 2) = 0$, then the two possible values of $5x + 2$ are

(A) -4 and $\frac{2}{5}$ **(B)** 0 and -18 **(C)** 0 and 22 **(D)** 0 and 4 **(E)** 4 and 22

16. In the diagram, circles C_1 and C_2 each have center O. The area of the shaded region is



(A) 2π **(B)** 3π **(C)** 4π **(D)** 6π **(E)** 8π

17. A cylinder with radius 2 cm and height 8 cm is full of water. A second cylinder of radius 4 cm and height 8 cm is empty. If all of the water is poured from the first cylinder into the second cylinder, the depth of the water in the second cylinder will be

(A) 1 cm **(B)** 2 cm **(C)** 3 cm **(D)** 4 cm **(E)** 6 cm

18. A test has ten questions. Points are awarded as follows:
- Each correct answer is worth 3 points.
- Each unanswered question is worth 1 point.
- Each incorrect answer is worth 0 points.

A total score that is *not* possible is

(A) 11 **(B)** 13 **(C)** 17 **(D)** 23 **(E)** 29

19. Sam bicycles at 16 km/h and Chris bicycles at 24 km/h. At noon, Sam is 1 km north of Chris, and each begins to ride north. How many minutes will it take for Chris to catch Sam?

(A) $1\frac{1}{2}$ **(B)** $2\frac{1}{2}$ **(C)** $3\frac{3}{4}$ **(D)** $7\frac{1}{2}$ **(E)** 8

20. In triangle ABC, if $AB = AC = x + 1$ and $BC = 2x - 2$, where $x > 1$, then the area of the triangle is always equal to

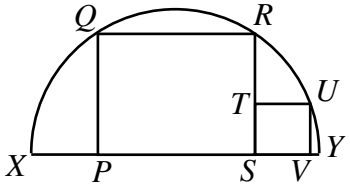
(A) $(x-1)\sqrt{2x^2+2}$ **(B)** $2(x-1)$ **(C)** $\frac{1}{2}(x+1)^2$ **(D)** $(x+1)(x-1)$ **(E)** $2(x-1)\sqrt{x}$

Part C: Each correct answer is worth 8.

21. Four *different* numbers a, b, c, and d are chosen from the list -1, -2, -3, -4, and -5. The largest possible value for the expression $a^b + c^d$ is

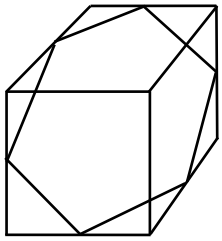
(A) $\frac{5}{4}$ **(B)** $\frac{7}{8}$ **(C)** $\frac{31}{32}$ **(D)** $\frac{10}{9}$ **(E)** $\frac{26}{25}$

22. In the diagram, a semi-circle has diameter XY. Rectangle $PQRS$ is inscribed in the semi-circle with $PQ = 12$ and $QR = 28$. Square $STUV$ has T on RS, U on the semi-circle and V on XY. The area of $STUV$ is closest to



(A) 12 **(B)** 13 **(C)** 16 **(D)** 14 **(E)** 15

23. A solid cube of side length 4 cm is cut into two pieces by a plane that passed through the midpoints of six edges, as shown. To the nearest square centimetre, the surface area of each half cube created is



(A) 69 **(B)** 48 **(C)** 32

(D) 65 **(E)** 58

24. The arithmetic sequence a, $a+d$, $a+2d$, $a+3d$, ..., $a+(n-1)d$ has the following properties:

- When the first, third, and fifth, and so on terms are added, up to and including the last term, the sum is 320.
- When the first, fourth, seventh, and so on, terms are added, up to and including the last term, the sum is 224.

What is the sum of the whole sequence?

(A) 656 **(B)** 640 **(C)** 608 **(D)** 704 **(E)** 672

25. A *triline* is a line with the property that three times its slope is equal to the sum of its x-intercept and its y-intercept. For how many integers q with $1 \leq q \leq 10\,000$ is there at least one positive integer p so that there is exactly one triline through (p, q)?

(A) 60 **(B)** 57 **(C)** 58 **(D)** 61 **(E)** 59

N.B. This problem has been corrected from its original version with the addition of the underlined word "positive".

Canadian Mathematics Competition

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Canadian
Mathematics
Competition

An activity of The Centre for Education
in Mathematics and Computing,
University of Waterloo, Waterloo, Ontario

Fermat Contest (Grade 11)

Wednesday, February 18, 2004

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Time: 1 hour

Calculators are permitted.

Instructions

1. Do not open the contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper right corner.
5. **Be certain that you code your name, age, sex, grade, and the contest you are writing on the response form. Only those who do so can be counted as official contestants.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. When you have decided on your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor instructs you to begin, you will have *sixty* minutes of working time.

Scoring:	There is *no penalty* for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. The value of $\dfrac{10}{10(11)-10^2}$ is

   (**A**) 100 (**B**) 2 (**C**) 10 (**D**) 1 (**E**) 11

2. $\sqrt{4^0+4^2+4^3}$ equals

   (**A**) 9 (**B**) 13 (**C**) 14 (**D**) 32 (**E**) 64

3. If $x=2-4+6$ and $y=1-3+5$, then $x-y$ equals

   (**A**) 0 (**B**) 1 (**C**) 5 (**D**) 3 (**E**) –1

4. A lemon loaf completely fills a pan measuring 20 cm by 18 cm by 5 cm. The loaf is cut into 25 pieces of equal volume. If the density of the loaf is $2\ \text{g/cm}^3$, how much does each of the 25 pieces weigh?

   (**A**) 72 g (**B**) 288 g (**C**) 36 g (**D**) 144 g (**E**) 720 g

5. If $\left(\dfrac{1}{2+3}\right)\left(\dfrac{1}{3+4}\right)=\dfrac{1}{x+5}$, the value of x is

   (**A**) 4 (**B**) 7 (**C**) 30 (**D**) 37 (**E**) 67

6. Three cans of juice fill $\frac{2}{3}$ of a one-litre jug. How many cans of juice are needed to completely fill 8 one-litre jugs?

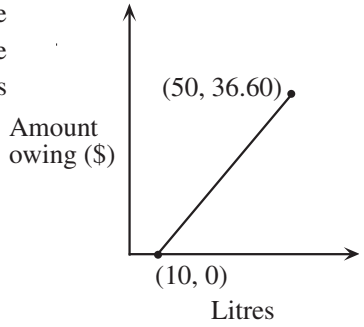
   (**A**) 36 (**B**) 12 (**C**) $\frac{16}{3}$ (**D**) 16 (**E**) 24

7. When $x=\frac{1}{5}$, the value of the expression $\dfrac{x^2-4}{x^2-2x}$ is

   (**A**) 0.4 (**B**) -0.52 (**C**) -5 (**D**) 10 (**E**) 11

8. Jane arrives at the Fermat Fuel Fill-up to fill up her gas tank. The graph shows the amount of gas that Jane had upon arrival, the amount that she purchased, and the cost of this purchase. What is the price per litre of the gas that she purchased?

   (**A**) 91.5¢ (**B**) 73.2¢ (**C**) 61.0¢
   (**D**) 53.2¢ (**E**) $1.09



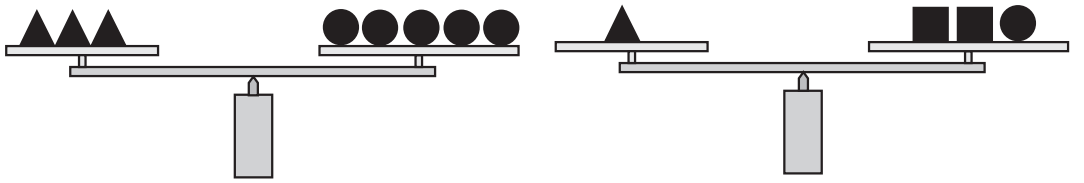
9. The table shows population information for two towns for the years 2003 and 2004.

Town	2003 Population	Percentage Change From 2003 to 2004
Cayleyville	10 000	4%
Pascalberg	25 000	–12%

What is the difference between the populations of the two towns in 2004?

(**A**) 12 400 (**B**) 11 600 (**C**) 17 600 (**D**) 13 800 (**E**) 17 400

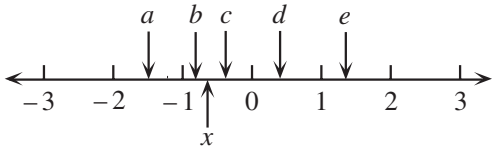
10. In the diagram, two equal-armed balances are shown. How many ■ would it take to balance one ●?



(**A**) 1 (**B**) 2 (**C**) 3 (**D**) 4 (**E**) 5

Part B: Each correct answer is worth 6.

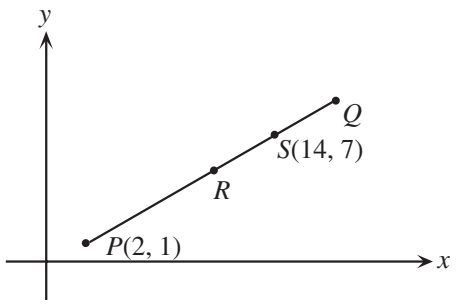
11. If x is located on the number line as shown, which letter best corresponds to the location of $-x^2$?



(**A**) a (**B**) b (**C**) c (**D**) d (**E**) e

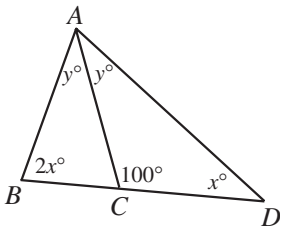
12. Point R is the midpoint of the line segment PQ and S is the midpoint of the line segment QR. If P has coordinates $(2, 1)$ and S has coordinates $(14, 7)$, then the coordinates of Q are

(**A**) $(8, 4)$ (**B**) $(26, 13)$ (**C**) $(10, 5)$
(**D**) $(18, 9)$ (**E**) $(16, 11)$

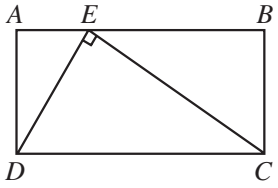


13. In the diagram, B, C and D lie on a straight line, with $\angle ACD = 100^\circ$, $\angle ADB = x^\circ$, $\angle ABD = 2x^\circ$, and $\angle DAC = \angle BAC = y^\circ$. The value of x is

(**A**) 10 (**B**) 45 (**C**) 30
(**D**) 50 (**E**) 20



14. In the diagram, $ABCD$ is a rectangle and point E lies on AB. Triangle DEC has $\angle DEC = 90°$, $DE = 3$ and $EC = 4$. The length of AD is

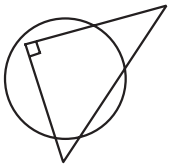


(**A**) 2.6 (**B**) 2.4 (**C**) 2.8
(**D**) 1.8 (**E**) 3.2

15. The graph of $x^2 - y^2 = 0$ is

(**A**) a straight line (**B**) a parabola (**C**) a circle
(**D**) a single point (**E**) two straight lines

16. A right-angled triangle has sides of length 6, 8 and 10. A circle is drawn so that the area inside the circle but outside this triangle equals the area inside the triangle but outside the circle. The radius of the circle is closest to



(**A**) 2.9 (**B**) 2.8 (**C**) 3.8
(**D**) 3.1 (**E**) 3.6

17. An increasing sequence is formed so that the difference between consecutive terms is a constant. If the first four terms of this sequence are x, y, $3x + y$, and $x + 2y + 2$, then the value of $y - x$ is

(**A**) 2 (**B**) 3 (**C**) 4 (**D**) 5 (**E**) 6

18. If $y = a(x-2)^2 + c$ and $y = (2x-5)(x-b)$ represent the same quadratic function, the value of b is

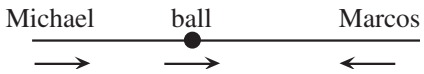
(**A**) 3 (**B**) $\frac{3}{2}$ (**C**) $\frac{4}{5}$ (**D**) $-\frac{5}{2}$ (**E**) $\frac{8}{5}$

19. A computer software retailer has 1200 copies of a new software package to sell. From past experience, she knows that:
- Half of them will sell right away at the original price she sets,
- Two-thirds of the remainder will sell later when their price is reduced by 40%, and
- The remaining copies will sell in a clearance sale at 75% off the original price.

In order to make a reasonable profit, the total sales revenue must be \$72 000. To the nearest cent, what original price should she set?

(**A**) \$60.01 (**B**) \$75.01 (**C**) \$79.13 (**D**) \$80.90 (**E**) \$240.01

20. A soccer ball rolls at 4 m/s towards Marcos in a direct line from Michael. The ball is 15 m ahead of Michael who is chasing it at 9 m/s. Marcos is 30 m away from the ball and is running towards it at 8 m/s. The distance between Michael and Marcos when the ball is touched for the first time by one of them is closest to



(**A**) 2.00 m (**B**) 2.25 m (**C**) 2.50 m
(**D**) 2.75 m (**E**) 3.00 m

continued ...

Part C: Each correct answer is worth 8.

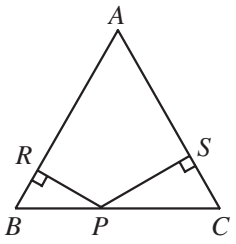
21. Bill and Jill are hired to paint a line on a road. If Bill works by himself, he could paint the line in B hours. If Jill works by herself, she could paint the line in J hours. Bill starts painting the line from one end, and Jill begins painting the line from the other end one hour later. They both work until the line is painted. Which is the following is an expression for the number of hours that Bill works?

(**A**) $\dfrac{B(J+1)}{B+J}$ (**B**) $J+1$ (**C**) $\dfrac{BJ}{B+J}+1$ (**D**) $\dfrac{B+J-1}{2}$ (**E**) $\dfrac{B(J-1)}{B+J}$

22. If k is the smallest positive integer such that $\left(2^k\right)\left(5^{300}\right)$ has 303 digits when expanded, then the sum of the digits of the expanded number is

(**A**) 11 (**B**) 10 (**C**) 8 (**D**) 7 (**E**) 5

23. Triangle ABC is isosceles with $AB = AC$ and $BC = 65$ cm. P is a point on BC such that the perpendicular distances from P to AB and AC are 24 cm and 36 cm, respectively. The area of ΔABC, in cm^2, is

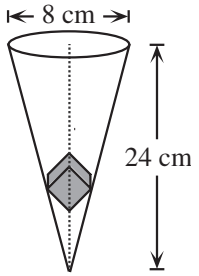


(**A**) 1254 (**B**) 1640 (**C**) 1950
(**D**) 2535 (**E**) 2942

24. The polynomial $f(x)$ satisfies the equation $f(x)-f(x-2)=(2x-1)^2$ for all x. If p and q are the coefficients of x^2 and x, respectively, in $f(x)$, then $p+q$ is equal to

(**A**) 0 (**B**) $\frac{5}{6}$ (**C**) $\frac{4}{3}$ (**D**) 1 (**E**) $\frac{2}{3}$

25. A steel cube has edges of length 3 cm, and a cone has a diameter of 8 cm and a height of 24 cm. The cube is placed in the cone so that one of its interior diagonals coincides with the axis of the cone. What is the distance, in cm, between the vertex of the cone and the closest vertex of the cube?



(**A**) $6\sqrt{6}-\sqrt{3}$ (**B**) $\dfrac{12\sqrt{6}-3\sqrt{3}}{4}$ (**C**) $6\sqrt{6}-2\sqrt{3}$
(**D**) $5\sqrt{3}$ (**E**) $6\sqrt{6}$

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PUBLICATIONS

Students and parents who enjoy solving problems for fun and recreation may find the following publications of interest. They are an excellent resource for enrichment, problem solving and contest preparation.

Copies of Previous Canadian Mathematics Competitions

Copies of previous contests and solutions are available at no cost in both English and French at http://www.cemc.uwaterloo.ca

Problems Problems Problems Books

Each volume is a collection of problems (multiple choice and full solution), grouped into 9 or more topics. Questions are selected from previous Canadian Mathematics Competition contests, and full solutions are provided for all questions. The price is $15. **(Available in English only.)**

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- over 200 problems and full solutions
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- 11 topics
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Include your name, address (with postal code), and telephone number.

Cheques or money orders in Canadian funds should be made payable to "Centre for Education in Mathematics and Computing". In Canada, add $3.00 for the first item ordered for shipping and handling, plus $1.00 for each subsequent item. No Provincial Sales Tax is required, but 7% GST must be added. Orders ***outside of Canada ONLY***, add $10.00 for the first item ordered for shipping and handling, plus $2.00 for each subsequent item. **Prices for these publications will remain in effect until September 1, 2004.**

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An activity of The Centre for Education in Mathematics and Computing, University of Waterloo, Waterloo, Ontario

Fermat Contest (Grade 11)

Wednesday, February 19, 2003

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Scoring:	There is *no penalty* for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. The value of $3^3 - 3^2 + 3^1 - 3^0$ is

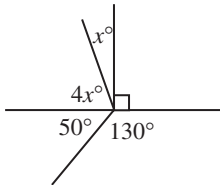
   (**A**) 18 (**B**) 6 (**C**) 9 (**D**) 40 (**E**) 20

2. If $a = 5$ and $a^2 + ab = 60$, the value of b is

   (**A**) 7 (**B**) 4 (**C**) 2 (**D**) 10 (**E**) 30

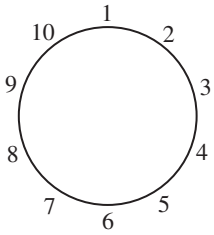
3. In the diagram, the value of x is

   (**A**) 22.5 (**B**) 25 (**C**) 20
   (**D**) 36 (**E**) 18



4. In the diagram, the numbers 1 to 10 are placed around a circle. Sandy crosses out 1, then 4, and then 7. Continuing in a clockwise direction, she crosses out every third number of those remaining, until only two numbers are left. The sum of these two numbers is

   (**A**) 13 (**B**) 10 (**C**) 8
   (**D**) 14 (**E**) 17



5. During hibernation, a bear loses 20% of its original mass. After hibernation, its mass is 220 kg. What was its mass, in kilograms, just before hibernation?

   (**A**) 176 (**B**) 264 (**C**) 240 (**D**) 275 (**E**) 1100

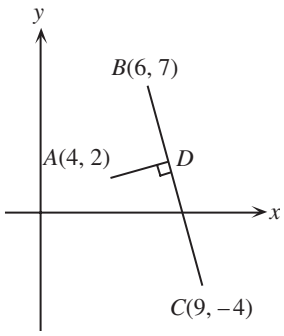
6. There are 2 girls and 6 boys playing a game. How many additional girls must join the game so that $\frac{5}{8}$ of the players are girls?

   (**A**) 6 (**B**) 3 (**C**) 5 (**D**) 8 (**E**) 7

7. A fish tank, in the shape of a rectangular prism, has a base measuring 20 cm by 40 cm with a height of 30 cm. The tank sits on a horizontal table and is half full of water. If 4000 cm^3 of water is added, what is the new depth of the water?

   (**A**) 5 cm (**B**) 15 cm (**C**) 20 cm (**D**) 25 cm (**E**) 10 cm

8. In the diagram, D is the point on BC so that AD is perpendicular to BC. The slope of AD is



(**A**) $\frac{3}{11}$ (**B**) 1 (**C**) $-\frac{15}{11}$

(**D**) $\frac{2}{7}$ (**E**) $\frac{2}{5}$

9. The average (mean) of $\frac{1}{5}$ and $\frac{1}{10}$ is $\frac{1}{x}$. The value of x is

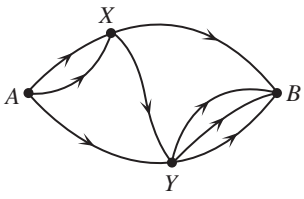
(**A**) $\frac{20}{3}$ (**B**) $\frac{3}{20}$ (**C**) 30 (**D**) $\frac{10}{3}$ (**E**) $\frac{2}{15}$

10. Carly takes three steps to walk the same distance as Jim walks in four steps. Each of Carly's steps covers 0.5 metres. How many metres does Jim travel in 24 steps?

(**A**) 16 (**B**) 9 (**C**) 36 (**D**) 12 (**E**) 18

Part B: Each correct answer is worth 6.

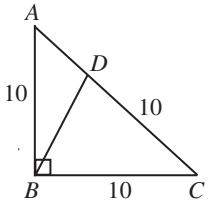
11. In the diagram, it is only possible to travel along an edge in the direction indicated by the arrow. Hazel studied the figure, and determined all the possible routes from A to B. She selected one of these routes at random. What is the probability that she selected a route which passes through X?



(**A**) $\frac{8}{11}$ (**B**) $\frac{3}{11}$ (**C**) 1

(**D**) $\frac{9}{11}$ (**E**) $\frac{6}{11}$

12. In the diagram, $\angle ABC = 90^\circ$ and $AB = BC = CD = 10$. The length of AD is closest to

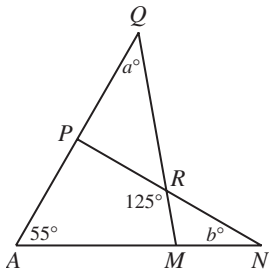


(**A**) 14 (**B**) 5 (**C**) 9
(**D**) 10 (**E**) 4

13. If $x + y = 1$ and $x - y = 3$, what is the value of $2^{x^2 - y^2}$?

(**A**) 4 (**B**) 8 (**C**) 2 (**D**) 16 (**E**) 32

14. In the diagram, AMN, APQ, QRM, and PRN are all straight lines. The value of $a+b$ is



(**A**) 70 (**B**) 55 (**C**) 80
(**D**) 90 (**E**) 75

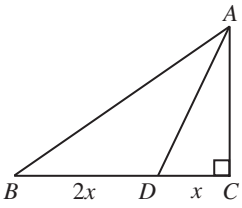
15. The side lengths of an equilateral triangle and a square are integers. If the triangle and the square have the same perimeter, which of the following is a possible side length of the triangle?

(**A**) 1 (**B**) 10 (**C**) 18 (**D**) 20 (**E**) 25

16. The product of the digits of a four-digit number is 810. If none of the digits is repeated, the sum of the digits is

(**A**) 18 (**B**) 19 (**C**) 23 (**D**) 25 (**E**) 22

17. In the diagram, ΔABC is right-angled at C. If $BD = 2x$, $DC = x$, and $\angle ADC = 2(\angle ABC)$, then the length of AB is

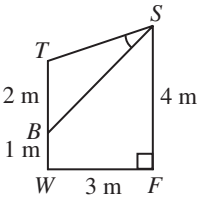


(**A**) $2\sqrt{2}x$ (**B**) $\sqrt{6}x$ (**C**) $2\sqrt{3}x$
(**D**) $3x$ (**E**) $4x$

18. A car uses 8.4 litres of gas for every 100 km it is driven. A mechanic is able to modify the car's engine at a cost of \$400 so that it will only use 6.3 litres of gas per 100 km. The owner determines the minimum distance that she would have to drive to recover the cost of the modifications. If gas costs \$0.80 per litre, this distance, in kilometres, is between

(**A**) 10 000 and 14 000 (**B**) 14 000 and 18 000 (**C**) 18 000 and 22 000
(**D**) 22 000 and 26 000 (**E**) 26 000 and 30 000

19. In an art gallery, a 2 m high painting, BT, is mounted on a wall with its bottom edge 1 m above the floor. A spotlight is mounted at S, 3 m out from the wall and 4 m above the floor. The size of $\angle TSB$ is closest to



(**A**) 27° (**B**) 63° (**C**) 34°
(**D**) 45° (**E**) 18°

20. If a, b and c are positive, consecutive terms of a geometric sequence (that is, $\frac{c}{b} = \frac{b}{a}$), then the graph of $y = ax^2 + bx + c$ is

(**A**) a curve that intersects the x-axis at two distinct points
(**B**) entirely below the x-axis
(**C**) entirely above the x-axis
(**D**) a straight line
(**E**) tangent to the x-axis

continued ...

Part C: Each correct answer is worth 8.

21. A sequence of numbers has 6 as its first term, and every term after the first is defined as follows: If a term, t, is even, the next term in the sequence is $\frac{1}{2}t$. If a term, s, is odd, the next term is $3s+1$. Thus, the first four terms in the sequence are 6, 3, 10, 5. The 100th term is

(**A**) 1 (**B**) 2 (**C**) 3 (**D**) 4 (**E**) 6

22. Pentagon *ABCDE* is such that all five diagonals *AC*, *BD*, *CE*, *DA,* and *EB* lie entirely within the pentagon. If the area of each of the triangles *ABC*, *BCD*, *CDE*, *DEA*, and *EAB* is equal to 1, the area of the pentagon *ABCDE* is closest to

(**A**) 3.62 (**B**) 3.64 (**C**) 3.66 (**D**) 3.68 (**E**) 3.70

23. Three faces of a rectangular box meet at a corner of the box. The centres of these faces form the vertices of a triangle having side lengths of 4 cm, 5 cm and 6 cm. The volume of the box, in cm^3, is

(**A**) $45\sqrt{3}$ (**B**) $45\sqrt{6}$ (**C**) $90\sqrt{6}$ (**D**) 125 (**E**) $120\sqrt{2}$

24. When the expression $\left[(1+x)\left(1+2x^3\right)\left(1+4x^9\right)\left(1+8x^{27}\right)\left(1+16x^{81}\right)\left(1+32x^{243}\right)\left(1+64x^{729}\right)\right]^2$ is expanded and simplified, the coefficient of x^{2003} is

(**A**) 0 (**B**) 2^{28} (**C**) 2^{30} (**D**) 2^{29} (**E**) 2^{31}

25. The set $\{1,4,n\}$ has the property that when any two distinct elements are chosen and 2112 is added to their product, the result is a perfect square. If n is a positive integer, the number of possible values for n is

(**A**) 8 (**B**) 7 (**C**) 6 (**D**) 5 (**E**) 4

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PUBLICATIONS

Students and parents who enjoy solving problems for fun and recreation may find the following publications of interest. They are an excellent resource for enrichment, problem solving and contest preparation.

Copies of Previous Canadian Mathematics Competitions

Copies of previous contests and solutions are available at no cost in both English and French at http://www.cemc.uwaterloo.ca

Problems Problems Problems Books

Each volume is a collection of problems (multiple choice and full solution), grouped into 9 or more topics. Questions are selected from previous Canadian Mathematics Competition contests, and full solutions are provided for all questions. The price is $15. **(Available in English only.)**

Volume 1
- over 300 problems and full solutions
- 10 topics
- for students in Grades 9, 10, & 11
- French version of Volume 1 is available

Volume 2
- over 325 problems and full solutions
- 10 topics (different from Volume 1)
- for students in Grades 9, 10, & 11

Volume 3
- over 235 problems and full solutions
- 12 topics
- for senior high school students

Volume 4
- over 325 problems and full solutions
- 12 topics
- for students in Grades 7, 8, & 9

Volume 5
- over 200 problems and full solutions
- 9 topics (different from Volume 3)
- for senior high school students

Volume 6
- over 300 problems and full solutions
- 11 topics
- for students in Grades 7, 8, & 9

Volume 7


- over 300 problems and full solutions
- 12 topics
- for students in Grades 9 and 10

Volume 8


- over 200 problems and full solutions
- 10 topics
- for students in Grades 11 and 12

Problems and How To Solve Them - Volume 1

This book continues the collection of problems available for enrichment of students in grades 9, 10, and 11. Included for each of the eight chapters is a discussion on solving problems, with suggested approaches. There are more than 225 new problems, almost all from Canadian Mathematics Competitions, with complete solutions. The price is $20. **(Available in English only.)**

Orders should be addressed to: Canadian Mathematics Competition
Faculty of Mathematics, Room 5181
University of Waterloo
Waterloo, ON N2L 3G1

Include your name, address (with postal code), and telephone number.

Cheques or money orders in Canadian funds should be made payable to "Centre for Education in Mathematics and Computing". In Canada, add $3.00 for the first item ordered for shipping and handling, plus $1.00 for each subsequent item. No Provincial Sales Tax is required, but 7% GST must be added. Orders ***outside of Canada ONLY***, add $10.00 for the first item ordered for shipping and handling, plus $2.00 for each subsequent item. **Prices for these publications will remain in effect until September 1, 2003.**

Canadian
Mathematics
Competition

An activity of The Centre for Education in Mathematics and Computing, University of Waterloo, Waterloo, Ontario

Fermat Contest (Grade 11)

Wednesday, February 20, 2002

Time: 1 hour

Calculators are permitted, providing they are non-programmable and without graphic displays.

Instructions

1. Do not open the contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper right corner.
5. **Be certain that you code your name, age, sex, grade, and the contest you are writing on the response form. Only those who do so can be counted as official contestants.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. When you have decided on your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor instructs you to begin, you will have *sixty* minutes of working time.

Scoring:	There is *no penalty* for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. If $x = 3$, the numerical value of $5 - 2x^2$ is

   (**A**) –1 (**B**) 27 (**C**) –13 (**D**) – 31 (**E**) 3

2. $\dfrac{3^3 + 3}{2^2 + 2}$ is equal to

   (**A**) 3 (**B**) 6 (**C**) 2 (**D**) $\frac{3}{2}$ (**E**) 5

3. If it is now 9:04 a.m., in 56 hours the time will be

   (**A**) 9:04 a.m. (**B**) 5:04 p.m. (**C**) 5:04 a.m. (**D**) 1:04 p.m. (**E**) 1:04 a.m.

4. Which one of the following statements is **not** true?
   (**A**) 25 is a perfect square.
   (**B**) 31 is a prime number.
   (**C**) 3 is the smallest prime number.
   (**D**) 8 is a perfect cube.
   (**E**) 15 is the product of two prime numbers.

5. A rectangular picture of Pierre de Fermat, measuring 20 cm by 40 cm, is positioned as shown on a rectangular poster measuring 50 cm by 100 cm. What percentage of the area of the poster is covered by the picture?

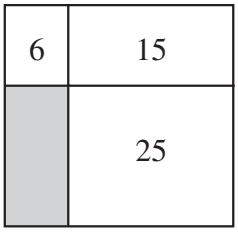
   

   (**A**) 24% (**B**) 16% (**C**) 20%
   (**D**) 25% (**E**) 40%

6. Gisa is taller than Henry but shorter than Justina. Ivan is taller than Katie but shorter than Gisa. The tallest of these five people is

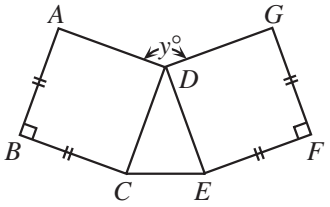
   (**A**) Gisa (**B**) Henry (**C**) Ivan (**D**) Justina (**E**) Katie

7. A rectangle is divided into four smaller rectangles. The areas of three of these rectangles are 6, 15 and 25, as shown. The area of the shaded rectangle is

   (**A**) 7 (**B**) 15 (**C**) 12
   (**D**) 16 (**E**) 10

8. In the diagram, $ABCD$ and $DEFG$ are squares with equal side lengths, and $\angle DCE = 70°$. The value of y is

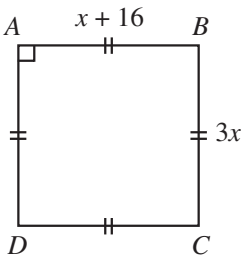


(**A**) 120 (**B**) 160 (**C**) 130
(**D**) 110 (**E**) 140

9. The numbers 1 through 20 are written on twenty golf balls, with one number on each ball. The golf balls are placed in a box, and one ball is drawn at random. If each ball is equally likely to be drawn, what is the probability that the number on the golf ball drawn is a multiple of 3?

(**A**) $\frac{3}{20}$ (**B**) $\frac{6}{20}$ (**C**) $\frac{10}{20}$ (**D**) $\frac{5}{20}$ (**E**) $\frac{1}{20}$

10. $ABCD$ is a square with $AB = x + 16$ and $BC = 3x$, as shown. The perimeter of $ABCD$ is



(**A**) 16 (**B**) 32 (**C**) 96
(**D**) 48 (**E**) 24

Part B: Each correct answer is worth 6.

11. A line passing through the points $(0,-2)$ and $(1,0)$ also passes through the point $(7,b)$. The numerical value of b is

(**A**) 12 (**B**) $\frac{9}{2}$ (**C**) 10 (**D**) 5 (**E**) 14

12. How many three-digit positive integers are perfect squares?

(**A**) 23 (**B**) 22 (**C**) 21 (**D**) 20 (**E**) 19

13. A "double-single" number is a three-digit number made up of two identical digits followed by a different digit. For example, 553 is a double-single number. How many double-single numbers are there between 100 and 1000?

(**A**) 81 (**B**) 18 (**C**) 72 (**D**) 64 (**E**) 90

14. The natural numbers from 1 to 2100 are entered sequentially in 7 columns, with the first 3 rows as shown. The number 2002 occurs in column m and row n. The value of $m+n$ is

	Column 1	Column 2	Column 3	Column 4	Column 5	Column 6	Column 7
Row 1	1	2	3	4	5	6	7
Row 2	8	9	10	11	12	13	14
Row 3	15	16	17	18	19	20	21
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

(**A**) 290 (**B**) 291 (**C**) 292 (**D**) 293 (**E**) 294

15. In a sequence of positive numbers, each term after the first two terms is the sum of *all of the previous terms*. If the first term is a, the second term is 2, and the sixth term is 56, then the value of a is

(**A**) 1 (**B**) 2 (**C**) 3 (**D**) 4 (**E**) 5

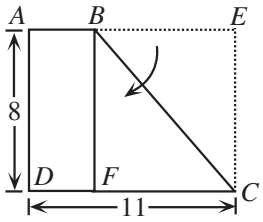
16. If $ac + ad + bc + bd = 68$ and $c + d = 4$, what is the value of $a + b + c + d$?

(**A**) 17 (**B**) 85 (**C**) 4 (**D**) 21 (**E**) 64

17. The average age of a group of 140 people is 24. If the average age of the males in the group is 21 and the average age of the females is 28, how many females are in the group?

(**A**) 90 (**B**) 80 (**C**) 70 (**D**) 60 (**E**) 50

18. A rectangular piece of paper $AECD$ has dimensions 8 cm by 11 cm. Corner E is folded onto point F, which lies on DC, as shown. The perimeter of trapezoid $ABCD$ is closest to

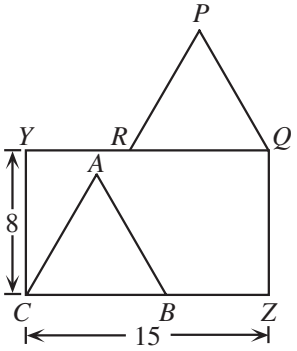


(**A**) 33.3 cm (**B**) 30.3 cm (**C**) 30.0 cm
(**D**) 41.3 cm (**E**) 35.6 cm

19. If $2^a 3^b = 8\left(6^{10}\right)$, where a and b are integers, then $b - a$ equals

(**A**) 0 (**B**) 23 (**C**) -13 (**D**) -7 (**E**) -3

20. In the diagram, $YQZC$ is a rectangle with $YC = 8$ and $CZ = 15$. Equilateral triangles ABC and PQR, each with side length 9, are positioned as shown with R and B on sides YQ and CZ, respectively. The length of AP is



(**A**) 10 (**B**) $\sqrt{117}$ (**C**) 9
(**D**) 8 (**E**) $\sqrt{72}$

Part C: Each correct answer is worth 8.

21. If $\sqrt{\frac{3}{1}\cdot\frac{5}{3}\cdot\frac{7}{5}\cdot\cdots\cdot\frac{2n+1}{2n-1}} = 9$, then the value of n is

(**A**) 38 (**B**) 1 (**C**) 40 (**D**) 4 (**E**) 39

22. The function $f(x)$ has the property that $f(x+y) = f(x) + f(y) + 2xy$, for all positive integers x and y. If $f(1) = 4$, then the numerical value of $f(8)$ is

(**A**) 72 (**B**) 84 (**C**) 88 (**D**) 64 (**E**) 80

continued ...

23. The integers from 1 to 9 are listed on a blackboard. If an additional m eights and k nines are added to the list, the average of all of the numbers in the list is 7.3. The value of $k+m$ is

(**A**) 24 (**B**) 21 (**C**) 11 (**D**) 31 (**E**) 89

24. A student has two open-topped cylindrical containers. (The walls of the two containers are thin enough so that their width can be ignored.) The larger container has a height of 20 cm, a radius of 6 cm and contains water to a depth of 17 cm. The smaller container has a height of 18 cm, a radius of 5 cm and is empty. The student slowly lowers the smaller container into the larger container, as shown in the cross-section of the cylinders in Figure 1. As the smaller container is lowered, the water first overflows out of the larger container (Figure 2) and then eventually pours into the smaller container. When the smaller container is resting on the bottom of the larger container, the depth of the water in the smaller container will be closest to

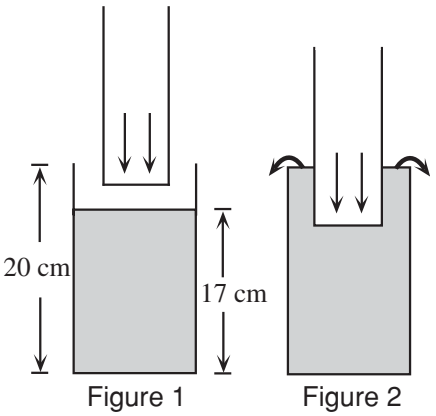


Figure 1 Figure 2

(**A**) 2.82 cm (**B**) 2.84 cm (**C**) 2.86 cm
(**D**) 2.88 cm (**E**) 2.90 cm

25. The lengths of all six edges of a tetrahedron are integers. The lengths of five of the edges are 14, 20, 40, 52, and 70. The number of possible lengths for the sixth edge is

(**A**) 9 (**B**) 3 (**C**) 4 (**D**) 5 (**E**) 6

Canadian Mathematics Competition

An activity of The Centre for Education in Mathematics and Computing, University of Waterloo, Waterloo, Ontario

Fermat Contest (Grade 11)

Wednesday, February 21, 2001

Time: 1 hour

Calculators are permitted, providing they are non-programmable and without graphic displays.

Instructions

1. Do not open the contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper right corner.
5. **Be certain that you code your name, age, sex, grade, and the contest you are writing on the response form. Only those who do so can be counted as official contestants.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. When you have decided on your choice, fill in the appropriate circles on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 20.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor instructs you to begin, you will have *sixty* minutes of working time.

Scoring:	There is *no penalty* for an incorrect answer. Each unanswered question is worth 2, to a maximum of 20.

Part A: Each correct answer is worth 5.

1. If $x+2x+3x+4x=5$, then x equals

(**A**) 10 (**B**) $\frac{1}{2}$ (**C**) $\frac{5}{4}$ (**D**) 2 (**E**) $\frac{5}{9}$

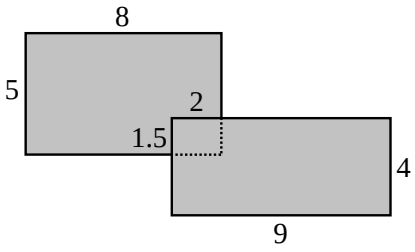
2. If $x=\frac{1}{4}$, which of the following has the largest value?

(**A**) x (**B**) x^2 (**C**) $\frac{1}{2}x$ (**D**) $\frac{1}{x}$ (**E**) $\sqrt{x}$

3. In a school, 30 boys and 20 girls entered the Fermat competition. Certificates were awarded to 10% of the boys and 20% of the girls. Of the students who participated, the percentage that received certificates was

(**A**) 14 (**B**) 15 (**C**) 16 (**D**) 30 (**E**) 50

4. Two rectangles overlap with their common region being a smaller rectangle, as shown. The total area of the shaded region is



(**A**) 45 (**B**) 70 (**C**) 52
(**D**) 79 (**E**) 73

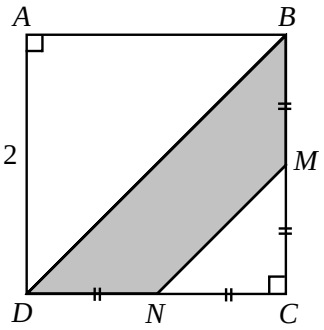
5. In ΔABC, $\angle A=3\angle B$ and $\angle B=2\angle C$. The measure of $\angle B$ is

(**A**) 10° (**B**) 20° (**C**) 30° (**D**) 40° (**E**) 60°

6. Pat gives half of his marbles to his best friend and then a third of those remaining to his sister. If his sister receives 9 marbles, then the number Pat keeps is

(**A**) 27 (**B**) 54 (**C**) 18 (**D**) 36 (**E**) 9

7. In the diagram, square $ABCD$ has side length 2, with M the midpoint of BC and N the midpoint of CD. The area of the shaded region $BMND$ is



(**A**) 1 (**B**) $2\sqrt{2}$ (**C**) $\frac{4}{3}$
(**D**) $\frac{3}{2}$ (**E**) $4-\frac{3}{2}\sqrt{2}$

8. If $\sqrt{5+11-7}=\sqrt{5+11}-\sqrt{x}$, then the value of x is

(**A**) 1 (**B**) 7 (**C**) -7 (**D**) 49 (**E**) 4

9. A bag contains 20 candies: 4 chocolate, 6 mint and 10 butterscotch. Candies are removed randomly from the bag and eaten. What is the minimum number of candies that must be removed to be *certain* that at least two candies of each flavour have been eaten?

(**A**) 6 (**B**) 10 (**C**) 12 (**D**) 16 (**E**) 18

10. When a positive integer N is divided by 60, the remainder is 49. When N is divided by 15, the remainder is

(**A**) 0 (**B**) 3 (**C**) 4 (**D**) 5 (**E**) 8

Part B: Each correct answer is worth 6.

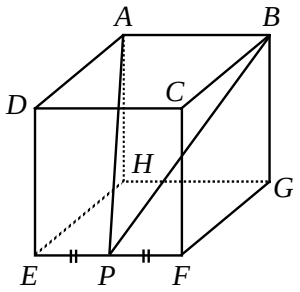
11. The fourth root of 2001200120012001 (that is, $\sqrt[4]{2001200120012001}$) is closest to

(**A**) 2 001 (**B**) 6 700 (**C**) 21 000 (**D**) 12 000 (**E**) 2 100

12. How many integer values of x satisfy $\frac{x-1}{3} < \frac{5}{7} < \frac{x+4}{5}$?

(**A**) 0 (**B**) 1 (**C**) 2 (**D**) 3 (**E**) 4

13. $ABCDEFGH$ is a cube having a side length of 2. P is the midpoint of EF, as shown. The area of ΔAPB is

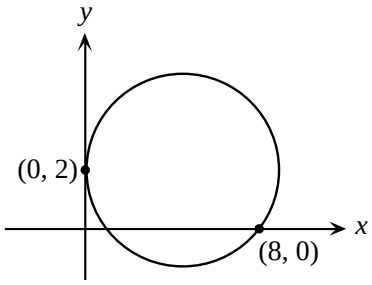


(A) $\sqrt{8}$ (**B**) 3 (**C**) $\sqrt{32}$
(**D**) $\sqrt{2}$ (**E**) 6

14. The last digit (that is, the units digit) of $(2002)^{2002}$ is

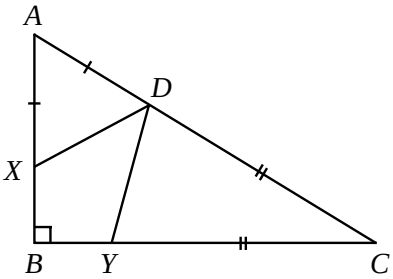
(**A**) 4 (**B**) 2 (**C**) 8 (**D**) 0 (**E**) 6

15. A circle is tangent to the y-axis at $(0, 2)$, and the larger of its x-intercepts is 8. The radius of the circle is



(**A**) $\frac{9}{2}$ (**B**) $\sqrt{17}$ (**C**) $\frac{17}{4}$
(**D**) $\frac{15}{4}$ (**E**) $\frac{\sqrt{17}}{2}$

16. In right triangle ABC, $AX = AD$ and $CY = CD$, as shown. The measure of $\angle XDY$ is

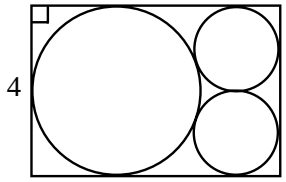


(**A**) 35° (**B**) 40° (**C**) 45°
(**D**) 50° (**E**) not determined by this information

17. Three different numbers are chosen such that when each of the numbers is added to the average of the remaining two, the numbers 65, 69 and 76 result. The average of the three original numbers is

(**A**) 34 (**B**) 35 (**C**) 36 (**D**) 37 (**E**) 38

18. In the diagram, the two smaller circles have equal radii. Each of the three circles is tangent to the other two circles, and each is also tangent to sides of the rectangle. If the width of the rectangle is 4, then its length is



(**A**) $2+\sqrt{8}$ (**B**) $3+\sqrt{8}$ (**C**) $3+\sqrt{10}$
(**D**) $\sqrt{32}$ (**E**) $4+\sqrt{3}$

19. Cindy leaves school at the same time every day. If she cycles at 20 km/h, she arrives home at 4:30 in the afternoon. If she cycles at 10 km/h, she arrives home at 5:15 in the afternoon. At what speed, in km/h, must she cycle to arrive home at 5:00 in the afternoon?

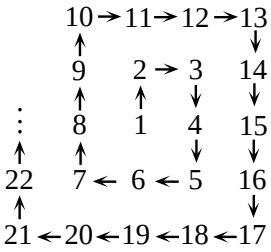
(**A**) $16\frac{2}{3}$ (**B**) 15 (**C**) $13\frac{1}{3}$ (**D**) 12 (**E**) $18\frac{3}{4}$

20. Point P is on the line $y=5x+3$. The coordinates of point Q are $(3,-2)$. If M is the midpoint of PQ, then M must lie on the line

(**A**) $y=\frac{5}{2}x-\frac{7}{2}$ (**B**) $y=5x+1$ (**C**) $y=-\frac{1}{5}x-\frac{7}{5}$ (**D**) $y=\frac{5}{2}x+\frac{1}{2}$ (**E**) $y=5x-7$

Part C: Each correct answer is worth 8.

21. A spiral of numbers is created, as shown, starting with 1. If the pattern of the spiral continues, in what configuration will the numbers 399, 400 and 401 appear?



(**A**) 399 → 400 → 401 (**B**) 401 ← 400 ← 399

(**C**)
401
↑
400
↑
399

(**D**)
399
↓
400
↓
401

(**E**)
400 → 401
↑
399

22. A sealed bottle, which contains water, has been constructed by attaching a cylinder of radius 1 cm to a cylinder of radius 3 cm, as shown in Figure A. When the bottle is right side up, the height of the water inside is 20 cm, as shown in the cross-section of the bottle in Figure B. When the bottle is upside down, the height of the liquid is 28 cm, as shown in Figure C. What is the total height, in cm, of the bottle?

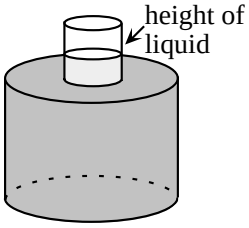


Figure A

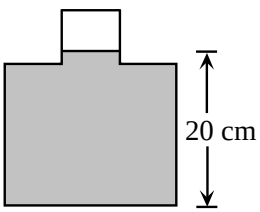


Figure B

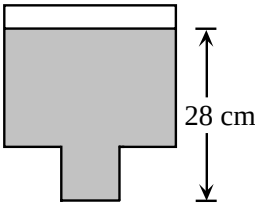


Figure C

(**A**) 29 (**B**) 30 (**C**) 31 (**D**) 32 (**E**) 48

continued ...

23. A sequence $t_1, t_2, ..., t_n, ...$ is defined as follows:

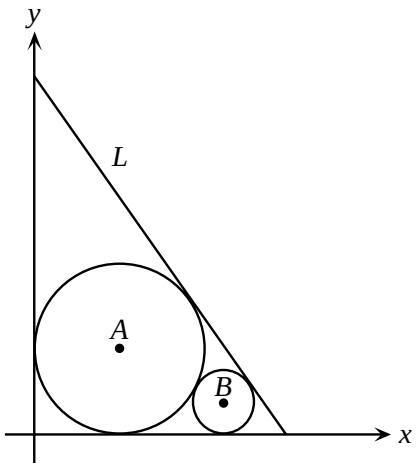
$t_1 = 14$

$t_k = 24 - 5t_{k-1}$, for each $k \geq 2$.

For every positive integer n, t_n can be expressed as $t_n = p \cdot q^n + r$, where p, q and r are constants. The value of $p + q + r$ is

(**A**) -5 (**B**) -3 (**C**) 3 (**D**) 17 (**E**) 31

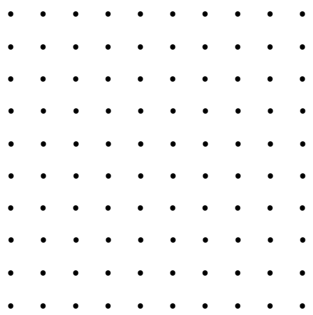
24. The circle with centre A has radius 3 and is tangent to both the positive x-axis and positive y-axis, as shown. Also, the circle with centre B has radius 1 and is tangent to both the positive x-axis and the circle with centre A. The line L is tangent to both circles. The y-intercept of line L is



(**A**) $3+6\sqrt{3}$ (**B**) $10+3\sqrt{2}$ (**C**) $8\sqrt{3}$

(**D**) $10+2\sqrt{3}$ (**E**) $9+3\sqrt{3}$

25. A square array of dots with 10 rows and 10 columns is given. Each dot is coloured either blue or red. Whenever two dots of the same colour are adjacent in the same row or column, they are joined by a line segment of the same colour as the dots. If they are adjacent but of different colours, they are then joined by a green line segment. In total, there are 52 red dots. There are 2 red dots at corners with an additional 16 red dots on the edges of the array. The remainder of the red dots are inside the array. There are 98 green line segments. The number of blue line segments is



(**A**) 36 (**B**) 37 (**C**) 38
(**D**) 39 (**E**) 40

Canadian
Mathematics
Competition

An activity of The Centre for Education in Mathematics and Computing, University of Waterloo, Waterloo, Ontario

Fermat Contest (Grade 11)

Wednesday, February 23, 2000

Time: 1 hour

Calculators are permitted, providing they are non-programmable and without graphic displays.

Instructions

1. Do not open the contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper right corner.
5. **Be certain that you code your name, age, sex, grade, and the contest you are writing on the response form. Only those who do so can be counted as official contestants.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. When you have decided on your choice, fill in the appropriate circles on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 20.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor instructs you to begin, you will have *sixty* minutes of working time.

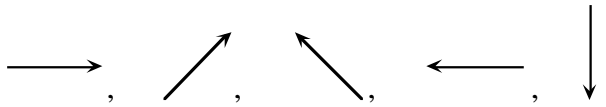
Scoring: There is *no penalty* for an incorrect answer.
Each unanswered question is worth 2 credits, to a maximum of 20 credits.

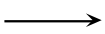
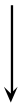
Part A: Each correct answer is worth 5.

1. The sum $29+12+23$ is equal to

   (**A**) 6^2 (**B**) 4^4 (**C**) 8^8 (**D**) 64^0 (**E**) 2^6

2. If the following sequence of five arrows repeats itself continuously, what arrow will be in the 48th position?



   (**A**)  (**B**)  (**C**)  (**D**) (**E**) 

3. A farmer has 7 cows, 8 sheep and 6 goats. How many more goats should be bought so that half of her animals will be goats?

   (**A**) 18 (**B**) 15 (**C**) 21 (**D**) 9 (**E**) 6

4. The square of 9 is divided by the cube root of 125. What is the remainder?

   (**A**) 6 (**B**) 3 (**C**) 16 (**D**) 2 (**E**) 1

5. The product of 2, 3, 5, and y is equal to its sum. What is the value of y?

   (**A**) $\frac{1}{3}$ (**B**) $\frac{10}{31}$ (**C**) $\frac{10}{29}$ (**D**) $\frac{3}{10}$ (**E**) $\frac{10}{3}$

6. A student uses a calculator to find an answer but instead of pressing the $\boxed{x^2}$ key presses the $\boxed{\sqrt{x}}$ key by mistake. The student's answer was 9. What should the answer have been?

   (**A**) 243 (**B**) 81 (**C**) 729 (**D**) 3 (**E**) 6561

7. The sum of the arithmetic series $(-300)+(-297)+(-294)+...+306+309$ is

   (**A**) 309 (**B**) 927 (**C**) 615 (**D**) 918 (**E**) 18

8. In a school referendum, $\frac{3}{5}$ of a student body voted 'yes' and 28% voted 'no'. If there were no spoiled ballots, what percentage of the students did not vote?

   (**A**) 72% (**B**) 40% (**C**) 32% (**D**) 12% (**E**) 88%

9. The numbers 6, 14, x, 17, 9, y, 10 have a mean of 13. What is the value of $x+y$?

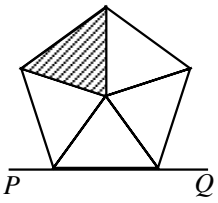
   (**A**) 20 (**B**) 21 (**C**) 23 (**D**) 25 (**E**) 35

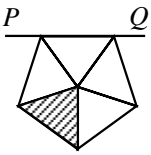
10. If $x\big(x(x+1)+2\big)+3 = x^3+x^2+x-6$ then x is equal to

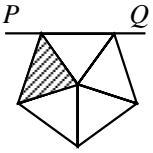
(**A**) 11 (**B**) -9 (**C**) -4 or 3 (**D**) -1 or 0 (**E**) -2

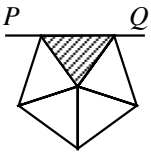
Part B: Each correct answer is worth 6.

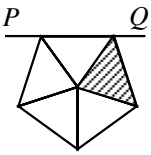
11. When the regular pentagon is reflected in the line PQ, and then rotated *clockwise* 144° about the centre of the pentagon, its position is

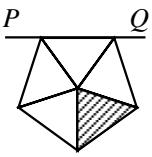


(**A**) 

(**B**) 

(**C**) 

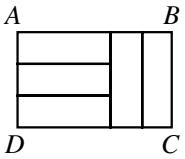
(**D**) 

(**E**) 

12. If the expression $15^6 \times 28^5 \times 55^7$ was evaluated, it would end with a string of consecutive zeros. How many zeros are in this string?

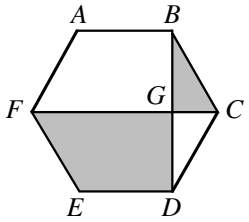
(**A**) 10 (**B**) 18 (**C**) 26 (**D**) 13 (**E**) 5

13. Rectangle $ABCD$ is divided into five congruent rectangles as shown. The ratio $AB:BC$ is



(**A**) 3:2 (**B**) 2:1 (**C**) 5:2
(**D**) 5:3 (**E**) 4:3

14. In the regular hexagon $ABCDEF$, two of the diagonals, FC and BD, intersect at G. The ratio of the area of quadrilateral $FEDG$ to the area of ΔBCG is



(**A**) $3\sqrt{3}:1$ (**B**) 4:1 (**C**) 6:1
(**D**) $2\sqrt{3}:1$ (**E**) 5:1

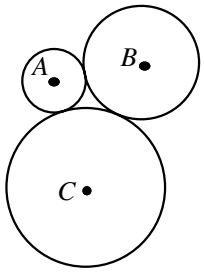
15. In a sequence, every term after the second term is twice the sum of the two preceding terms. The seventh term of the sequence is 8, and the ninth term is 24. What is the eleventh term of the sequence?

(**A**) 160 (**B**) 304 (**C**) 28 (**D**) 56 (**E**) 64

16. The digits 2, 2, 3, and 5 are randomly arranged to form a four digit number. What is the probability that the sum of the first and last digits is even?

(**A**) $\frac{1}{4}$ (**B**) $\frac{1}{3}$ (**C**) $\frac{1}{6}$ (**D**) $\frac{1}{2}$ (**E**) $\frac{2}{3}$

17. Three circles have centres A, B and C with radii 2, 4 and 6 respectively. The circles are tangent to each other as shown. Triangle ABC has



(**A**) $\angle A$ obtuse (**B**) $\angle B = 90°$ (**C**) $\angle A = 90°$

(**D**) all angles acute (**E**) $\angle B = \angle C$

18. If $P = 3^{2000} + 3^{-2000}$ and $Q = 3^{2000} - 3^{-2000}$ then the value of $P^2 - Q^2$ is

(**A**) 3^{4000} (**B**) 2×3^{-4000} (**C**) 0 (**D**) 2×3^{4000} (**E**) 4

19. An ant walks inside a 18 cm by 150 cm rectangle. The ant's path follows straight lines which always make angles of $45°$ to the sides of the rectangle. The ant starts from a point X on one of the shorter sides. The first time the ant reaches the opposite side, it arrives at the mid-point. What is the distance, in centimetres, from X to the nearest corner of the rectangle?

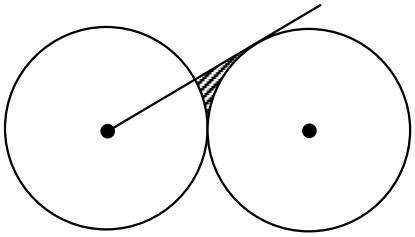
(**A**) 3 (**B**) 4 (**C**) 6 (**D**) 8 (**E**) 9

20. Given $a + 2b + 3c + 4d + 5e = k$ and $5a = 4b = 3c = 2d = e$, find the smallest positive integer value for k so that a, b, c, d, and e are all positive integers.

(**A**) 87 (**B**) 522 (**C**) 10 (**D**) 120 (**E**) 60

Part C: Each question is worth 8 credits.

21. Two circles of radius 10 are tangent to each other. A tangent is drawn from the centre of one of the circles to the second circle. To the nearest integer, what is the area of the shaded region?

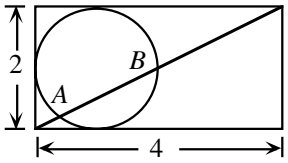


(**A**) 6 (**B**) 7 (**C**) 8
(**D**) 9 (**E**) 10

22. The left most digit of an integer of length 2000 digits is 3. In this integer, any two consecutive digits must be divisible by 17 or 23. The 2000th digit may be either 'a' or 'b'. What is the value of $a + b$?

(**A**) 3 (**B**) 7 (**C**) 4 (**D**) 10 (**E**) 17

23. A circle is tangent to three sides of a rectangle having side lengths 2 and 4 as shown. A diagonal of the rectangle intersects the circle at points A and B. The length of AB is



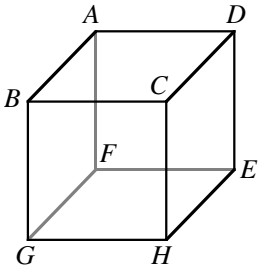
(**A**) $\sqrt{5}$ (**B**) $\frac{4\sqrt{5}}{5}$ (**C**) $\sqrt{5} - \frac{1}{5}$

(**D**) $\sqrt{5} - \frac{1}{6}$ (**E**) $\frac{5\sqrt{5}}{6}$

24. For the system of equations $x^2 + x^2y^2 + x^2y^4 = 525$ and $x + xy + xy^2 = 35$, the sum of the real y values that satisfy the equations is

(**A**) 20 (**B**) 2 (**C**) 5 (**D**) $\frac{55}{2}$ (**E**) $\frac{5}{2}$

25. The given cube is cut into four pieces by two planes. The first plane is parallel to face $ABCD$ and passes through the midpoint of edge BG. The second plane passes through the midpoints of edges AB, AD, HE, and GH. Determine the ratio of the volumes of the smallest and largest of the four pieces.



(**A**) 3:8 (**B**) 7:24 (**C**) 7:25
(**D**) 7:17 (**E**) 5:11

Canadian Mathematics Competition

An activity of The Centre for Education
in Mathematics and Computing,
University of Waterloo, Waterloo, Ontario

Fermat Contest (Grade 11)

Wednesday, February 24, 1999

Time: 1 hour

Calculators are permitted, providing they are non-programmable and without graphic displays.

Instructions

1. Do not open the contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
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5. **Be certain that you code your name, age, sex, grade, and the contest you are writing on the response form. Only those who do so can be counted as official contestants.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. When you have decided on your choice, fill in the appropriate circles on the response form.
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   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2 credits, to a maximum of 20 credits.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor instructs you to begin, you will have *sixty* minutes of working time.

Scoring: There is *no penalty* for an incorrect answer.
Each unanswered question is worth 2 credits, to a maximum of 20 credits.

Part A: Each question is worth 5 credits.

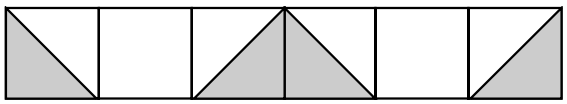
1. The value of $\left(\sqrt{25}-\sqrt{9}\right)^2$ is

   (**A**) 26 (**B**) 16 (**C**) 34 (**D**) 8 (**E**) 4

2. Today is Wednesday. What day of the week will it be 100 days from now?

   (**A**) Monday (**B**) Tuesday (**C**) Thursday (**D**) Friday (**E**) Saturday

3. Six squares are drawn and shaded as shown. What fraction of the total area is shaded?



   (**A**) $\frac{1}{2}$ (**B**) $\frac{1}{3}$ (**C**) $\frac{1}{4}$ (**D**) $\frac{2}{5}$ (**E**) $\frac{2}{3}$

4. Turning a screwdriver 90° will drive a screw 3 mm deeper into a piece of wood. How many complete revolutions are needed to drive the screw 36 mm into the wood?

   (**A**) 3 (**B**) 4 (**C**) 6 (**D**) 9 (**E**) 12

5. A value of x such that $(5-3x)^5=-1$ is

   (**A**) $\frac{4}{3}$ (**B**) 0 (**C**) $\frac{10}{3}$ (**D**) $\frac{5}{3}$ (**E**) 2

6. The number which is 6 less than twice the square of 4 is

   (**A**) -26 (**B**) 10 (**C**) 26 (**D**) 38 (**E**) 58

7. The Partridge family pays each of their five children a weekly allowance. The average allowance for each of the three younger children is \$8. The two older children each receive an average allowance of \$13. The total amount of allowance money paid per week is

   (**A**) \$50 (**B**) \$52.50 (**C**) \$105 (**D**) \$21 (**E**) \$55

8. The time on a digital clock is 5:55. How many minutes will pass before the clock next shows a time with all digits identical?

   (**A**) 71 (**B**) 72 (**C**) 255 (**D**) 316 (**E**) 436

9. In an election, Harold received 60% of the votes and Jacquie received all the rest. If Harold won by 24 votes, how many people voted?

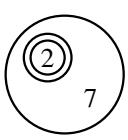
   (**A**) 40 (**B**) 60 (**C**) 72 (**D**) 100 (**E**) 120

10. If x and y are each chosen from the set $\{1, 2, 3, 5, 10\}$, the largest possible value of $\frac{x}{y}+\frac{y}{x}$ is

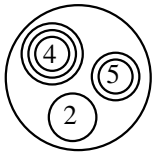
(**A**) 2 (**B**) $12\frac{1}{2}$ (**C**) $10\frac{1}{10}$ (**D**) $2\frac{1}{2}$ (**E**) 20

Part B: Each question is worth 6 credits.

11. In *Circle Land*, the numbers 207 and 4520 are shown in the following way:

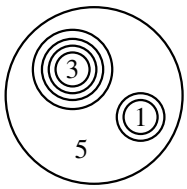


207



4520

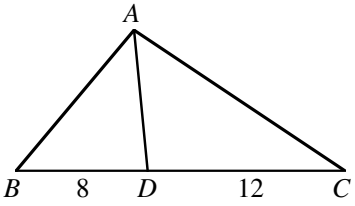
In *Circle Land*, what number does the following diagram represent?



(**A**) 30 105 (**B**) 30 150 (**C**) 3105 (**D**) 3015 (**E**) 315

12. The area of ΔABC is 60 square units. If $BD = 8$ units and $DC = 12$ units, the area (in square units) of ΔABD is

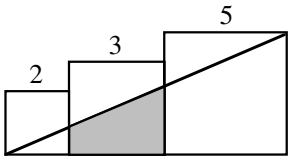
(**A**) 24 (**B**) 40 (**C**) 48
(**D**) 36 (**E**) 6



13. Crippin wrote four tests each with a maximum possible mark of 100. The average mark he obtained on these tests was 88. What is the lowest score he could have achieved on one of these tests?

(**A**) 88 (**B**) 22 (**C**) 52 (**D**) 0 (**E**) 50

14. Three squares have dimensions as indicated in the diagram. What is the area of the shaded quadrilateral?



(**A**) $\frac{21}{4}$ (**B**) $\frac{9}{2}$ (**C**) 5

(**D**) $\frac{15}{4}$ (**E**) $\frac{25}{4}$

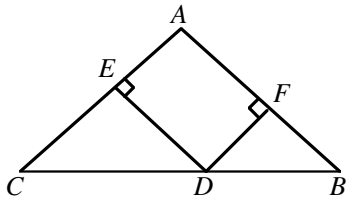
15. If $(a+b+c+d+e+f+g+h+i)^2$ is expanded and simplified, how many different terms are in the final answer?

(**A**) 36 (**B**) 9 (**C**) 45 (**D**) 81 (**E**) 72

16. If $px + 2y = 7$ and $3x + qy = 5$ represent the same straight line, then p equals

(**A**) 5 (**B**) 7 (**C**) 21 (**D**) $\frac{21}{5}$ (**E**) $\frac{10}{7}$

17. In ΔABC, $AC = AB = 25$ and $BC = 40$. D is a point chosen on BC. From D, perpendiculars are drawn to meet AC at E and AB at F. $DE + DF$ equals



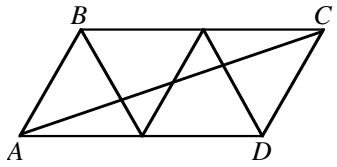
(**A**) 12 (**B**) 35 (**C**) 24

(**D**) 25 (**E**) $\frac{35}{2}\sqrt{2}$

18. The number of solutions (P, Q) of the equation $\frac{P}{Q} - \frac{Q}{P} = \frac{P+Q}{PQ}$, where P and Q are integers from 1 to 9 inclusive, is

(**A**) 1 (**B**) 8 (**C**) 16 (**D**) 72 (**E**) 81

19. Parallelogram $ABCD$ is made up of four equilateral triangles of side length 1. The length of diagonal AC is



(**A**) $\sqrt{5}$ (**B**) $\sqrt{7}$ (**C**) 3
(**D**) $\sqrt{3}$ (**E**) $\sqrt{10}$

20. If $a_1 = \frac{1}{1-x}$, $a_2 = \frac{1}{1-a_1}$, and $a_n = \frac{1}{1-a_{n-1}}$, for $n \geq 2$, $x \neq 1$ and $x \neq 0$, then a_{107} is

(**A**) $\frac{1}{1-x}$ (**B**) x (**C**) $-x$ (**D**) $\frac{x-1}{x}$ (**E**) $\frac{1}{x}$

Part C: Each question is worth 8 credits.

21. How many integers can be expressed as a sum of three distinct numbers if chosen from the set $\{4, 7, 10, 13, ..., 46\}$?

(**A**) 45 (**B**) 37 (**C**) 36 (**D**) 43 (**E**) 42

22. If $x^2 + ax + 48 = (x + y)(x + z)$ and $x^2 - 8x + c = (x + m)(x + n)$, where y, z, m, and n are integers between -50 and 50, then the maximum value of ac is

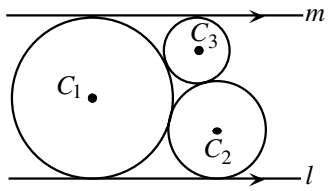
(**A**) 343 (**B**) 126 (**C**) 52 234 (**D**) 784 (**E**) 98 441

23. The sum of all values of x that satisfy the equation $\left(x^2 - 5x + 5\right)^{x^2+4x-60} = 1$ is

(**A**) -4 (**B**) 3 (**C**) 1 (**D**) 5 (**E**) 6

continued ...

24. Two circles C_1 and C_2 touch each other externally and the line l is a common tangent. The line m is parallel to l and touches the two circles C_1 and C_3. The three circles are mutually tangent. If the radius of C_2 is 9 and the radius of C_3 is 4, what is the radius of C_1?



(**A**) 10.4 (**B**) 11 (**C**) $8\sqrt{2}$
(**D**) 12 (**E**) $7\sqrt{3}$

25. Given that n is an integer, for how many values of n is $\dfrac{2n^2-10n-4}{n^2-4n+3}$ an integer?

(**A**) 9 (**B**) 7 (**C**) 6 (**D**) 4 (**E**) 5

35th Anniversary
1963 – 1998

Canadian Mathematics Competition

An activity of The Centre for Education in Mathematics and Computing, University of Waterloo, Waterloo, Ontario

Fermat Contest (Grade 11)

Wednesday, February 18, 1998

Time: 1 hour

Calculators are permitted, providing they are non-programmable and without graphic displays.

Instructions

1. Do not open the contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper right corner.
5. **Be certain that you code your name, age, sex, grade, and the contest you are writing on the response form. Only those who do so can be counted as official contestants.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. When you have decided on your choice, fill in the appropriate circles on the response form.
7. Scoring: Each correct answer is worth 5 credits in Part A, 6 credits in Part B, and 8 credits in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2 credits, to a maximum of 20 credits.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor instructs you to begin, you will have *sixty* minutes of working time.

Scoring: There is *no penalty* for an incorrect answer.
Each unanswered question is worth 2 credits, to a maximum of 20 credits.

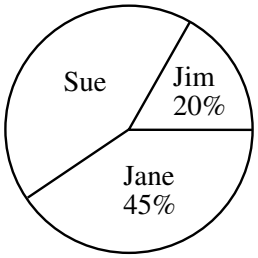
Part A: Each question is worth 5 credits.

1. The value of $\frac{1+2+3+4+5}{2+4+6+8+10}$ is

   (A) $\frac{1}{3}$ **(B)** 2.5 **(C)** $\frac{1}{2}$ **(D)** $\frac{11}{26}$ **(E)** $\frac{3}{8}$

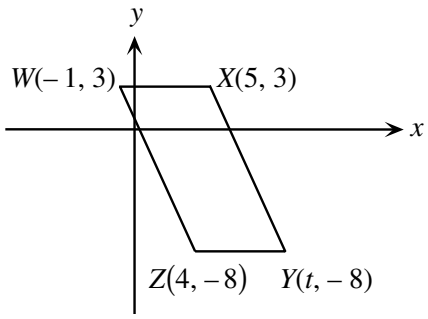
2. The pie chart shows a percentage breakdown of 1000 votes in a student election. How many votes did Sue receive?

   (A) 550 **(B)** 350 **(C)** 330
   (D) 450 **(E)** 935



3. If $WXYZ$ is a parallelogram, then t equals

   (A) 8 **(B)** 9 **(C)** 10
   (D) 11 **(E)** 12



4. The product of two positive integers p and q is 100. What is the largest possible value of $p+q$?

   (A) 52 **(B)** 101 **(C)** 20 **(D)** 29 **(E)** 25

5. If $\otimes$ is a new operation defined as $p \otimes q = p^2 - 2q$, what is the value of $7 \otimes 3$?

   (A) 43 **(B)** 8 **(C)** 141 **(D)** 36 **(E)** 26

6. The value of $\frac{1}{3}$ of 6^{30} is

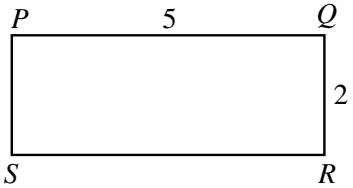
   (A) 6^{10} **(B)** 2^{30} **(C)** 2^{10} **(D)** 2×6^{29} **(E)** 2×6^{10}

7. The average (mean) of a list of 10 numbers is 0. If 72 and -12 are added to the list, the new average will be

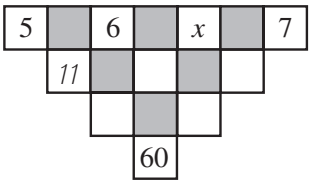
   (A) 30 **(B)** 6 **(C)** 0 **(D)** 60 **(E)** 5

8. On a rectangular table 5 units long and 2 units wide, a ball is rolled from point P at an angle of 45° to PQ and bounces off SR. The ball continues to bounce off the sides at 45° until it reaches S. How many bounces of the ball are required?

   (A) 9 **(B)** 8 **(C)** 7
   (D) 5 **(E)** 4

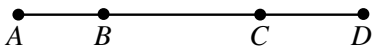


9. The number in an unshaded square is obtained by adding the numbers connected to it from the row above. (The '*11*' is one such number.) The value of x must be



(**A**) 4 (**B**) 6 (**C**) 9
(**D**) 15 (**E**) 10

10. Four points are on a line segment, as shown.

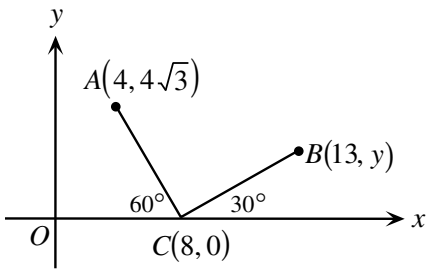


If $AB:BC=1:2$ and $BC:CD=8:5$, then $AB:BD$ equals
(**A**) $4:13$ (**B**) $1:13$ (**C**) $1:7$
(**D**) $3:13$ (**E**) $4:17$

Part B: Each question is worth 6 credits.

11. The number of solutions (x, y) of the equation $3x+y=100$, where x and y are positive integers, is
(**A**) 33 (**B**) 35 (**C**) 100 (**D**) 101 (**E**) 97

12. In the diagram, the value of y is
(**A**) $\frac{13}{2\sqrt{3}}$ (**B**) $\frac{5}{\sqrt{3}}$ (**C**) 2
(**D**) 12 (**E**) $\frac{\sqrt{3}}{5}$



13. Three-digit integers are formed using only the digits 1 and/or 2. The sum of all such integers formed is
(**A**) 1332 (**B**) 333 (**C**) 999 (**D**) 666 (**E**) 1665

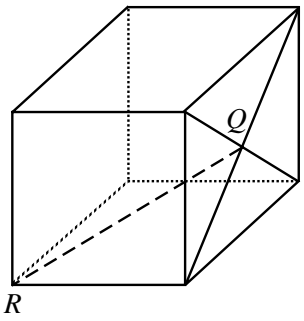
14. Three straight lines, l_1, l_2 and l_3, have slopes $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$, respectively. All three lines have the same y-intercept. If the sum of the x-intercepts of the three lines is 36, then the y-intercept is
(**A**) $\frac{-13}{12}$ (**B**) $\frac{-12}{13}$ (**C**) -4 (**D**) 4 (**E**) -9

15. If $-2 \le x \le 5$, $-3 \le y \le 7$, $4 \le z \le 8$, and $w = xy - z$, then the smallest value w may have is
(**A**) -14 (**B**) -18 (**C**) -19 (**D**) -22 (**E**) -23

16. If $N = \left(7^{p+4}\right)\left(5^{q}\right)\left(2^{3}\right)$ is a perfect cube, where p and q are positive integers, the smallest possible value of $p+q$ is
(**A**) 5 (**B**) 2 (**C**) 8 (**D**) 6 (**E**) 12

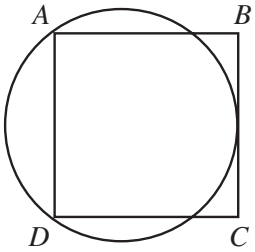
17. Using only digits 1, 2, 3, 4, and 5, a sequence is created as follows: one 1, two 2's, three 3's, four 4's, five 5's, six 1's, seven 2's, and so on.
The sequence appears as: 1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 5, 5, 1, 1, 1, 1, 1, 1, 2, 2, ...
The 100th digit in the sequence is
(**A**) 1 (**B**) 2 (**C**) 3 (**D**) 4 (**E**) 5

18. Q is the point of intersection of the diagonals of one face of a cube whose edges have length 2 units. The length of QR is



(**A**) 2 (**B**) $\sqrt{8}$ (**C**) $\sqrt{5}$
(**D**) $\sqrt{12}$ (**E**) $\sqrt{6}$

19. Square $ABCD$ has sides of length 14. A circle is drawn through A and D so that it is tangent to BC, as shown. What is the radius of the circle?



(**A**) 8.5 (**B**) 8.75 (**C**) 9
(**D**) 9.25 (**E**) 9.5

20. A deck of 100 cards is numbered from 1 to 100. Each card has the same number printed on both sides. One side of each card is red and the other side is yellow. Barsby places all the cards, red side up, on a table. He first turns over every card that has a number divisible by 2. He then examines all the cards, and turns over every card that has a number divisible by 3. How many cards have the red side up when Barsby is finished?

(**A**) 83 (**B**) 17 (**C**) 66 (**D**) 50 (**E**) 49

Part C: Each question is worth 8 credits.

21. The numbers 123 456 789 and 999 999 999 are multiplied. How many of the digits in the final result are 9's?

(**A**) 0 (**B**) 1 (**C**) 2 (**D**) 3 (**E**) 17

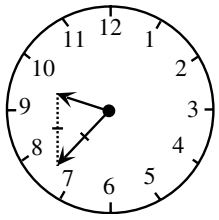
22. There are four unequal, positive integers a, b, c, and N such that $N = 5a + 3b + 5c$. It is also true that $N = 4a + 5b + 4c$ and N is between 131 and 150. What is the value of $a + b + c$?

(**A**) 13 (**B**) 17 (**C**) 22 (**D**) 33 (**E**) 36

23. Three rugs have a combined area of 200 m^2. By overlapping the rugs to cover a floor area of 140 m^2, the area which is covered by exactly two layers of rug is 24 m^2. What area of floor is covered by three layers of rug?

(**A**) 12 m^2 (**B**) 18 m^2 (**C**) 24 m^2 (**D**) 36 m^2 (**E**) 42 m^2

24. At some time between 9:30 and 10 o'clock the triangle determined by the minute hand and the hour hand is an isosceles triangle (see diagram). If the two equal angles in this triangle are each twice as large as the third angle, what is the time?



(**A**) 9:35 (**B**) 9:36 (**C**) 9:37
(**D**) 9:38 (**E**) 9:39

25. For each value of x, $f(x)$ is defined to be the minimum value of the three numbers $2x + 2$, $\frac{1}{2}x + 1$ and $-\frac{3}{4}x + 7$. What is the maximum value of $f(x)$?

(**A**) $\frac{2}{3}$ (**B**) 2 (**C**) $\frac{17}{5}$ (**D**) $\frac{62}{11}$ (**E**) 7

Canadian
Mathematics
Competition

An activity of The Centre for Education
in Mathematics and Computing,
University of Waterloo, Waterloo, Ontario

Fermat Contest (Grade 11)

Wednesday, February 19, 1997

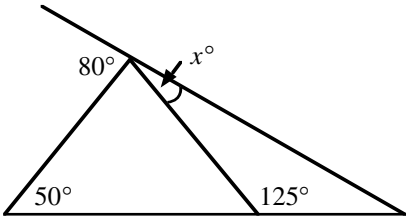
Part A: Each question is worth 5 credits.

1. The value of $(1)^{10}+(-1)^8+(-1)^7+(1)^5$ is

   (**A**) 0 (**B**) 4 (**C**) 1 (**D**) 16 (**E**) 2

2. The value of x is

   (**A**) 15 (**B**) 20 (**C**) 25
   (**D**) 30 (**E**) 35



3. The greatest number of Mondays that can occur in 45 consecutive days is

   (**A**) 5 (**B**) 6 (**C**) 7 (**D**) 8 (**E**) 9

4. The product of a positive number, its square, and its reciprocal is $\frac{100}{81}$. What is the number?

   (**A**) $\frac{81}{100}$ (**B**) $\frac{100}{81}$ (**C**) $\frac{9}{10}$ (**D**) $\frac{10}{9}$ (**E**) $\frac{10\,000}{6561}$

5. The sum of seven consecutive positive integers is 77. The largest of these integers is

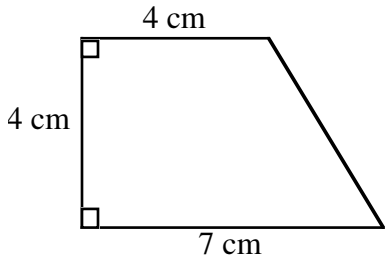
   (**A**) 8 (**B**) 11 (**C**) 14 (**D**) 15 (**E**) 17

6. If 2×10^3 is represented as $2E3$ on a certain calculator, how would the product of $2E3$ and $3E2$ be represented?

   (**A**) $6E6$ (**B**) $6E5$ (**C**) $5E5$ (**D**) $2.3E3$ (**E**) $5E6$

7. The perimeter of the figure shown is

   (**A**) 19 cm (**B**) 22 cm (**C**) 21 cm
   (**D**) 15 cm (**E**) 20 cm



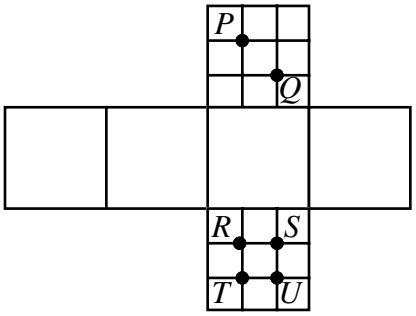
8. Three of the vertices of a parallelogram are $(0,1)$, $(1,2)$, and $(2,1)$. The area of the parallelogram is

   (**A**) 1 (**B**) 2 (**C**) $\sqrt{2}$ (**D**) $2\sqrt{2}$ (**E**) 4

9. If $10\le x\le 20$ and $40\le y\le 60$, the largest value of $\frac{x^2}{2y}$ is

   (**A**) 5 (**B**) $\frac{5}{6}$ (**C**) $\frac{10}{3}$ (**D**) $\frac{5}{4}$ (**E**) 10

10. On a cube, two points are said to be diametrically opposite if the line containing the two points also contains the centre of the cube. The diagram below shows a pattern which could be folded into a cube. Which point would be diametrically opposite to point P?



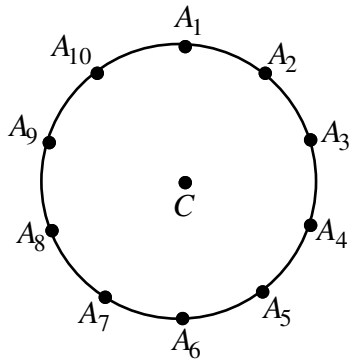
(**A**) Q (**B**) R (**C**) S
(**D**) T (**E**) U

Part B: Each question is worth 6 credits.

11. Five integers have an average of 69. The middle integer (the median) is 83. The most frequently occurring integer (the mode) is 85. The range of the five integers is 70. What is the second smallest of the five integers?

(**A**) 77 (**B**) 15 (**C**) 50 (**D**) 55 (**E**) 49

12. On a circle, ten points $A_1, A_2, A_3, \ldots, A_{10}$ are equally spaced. If C is the centre of the circle, what is the size, in degrees, of the angle A_1A_5C?

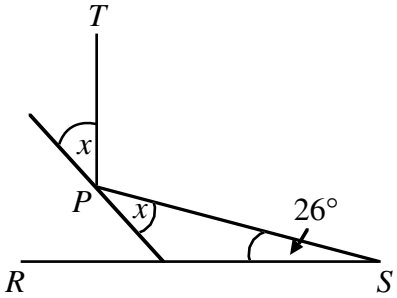


(**A**) 18 (**B**) 36 (**C**) 10
(**D**) 72 (**E**) 144

13. The digits 1, 2, 3, 4 can be arranged to form twenty-four different 4-digit numbers. If these twenty-four numbers are listed from smallest to largest, in what position is 3142?

(**A**) 13th (**B**) 14th (**C**) 15th (**D**) 16th (**E**) 17th

14. A beam of light shines from point S, reflects off a reflector at point P, and reaches point T so that PT is perpendicular to RS. Then x is



(**A**) 26° (**B**) 13° (**C**) 64°
(**D**) 32° (**E**) 58°

15. If $x^2yz^3 = 7^3$ and $xy^2 = 7^9$, then xyz equals

(**A**) 7^{10} (**B**) 7^9 (**C**) 7^8 (**D**) 7^6 (**E**) 7^4

16. The sum of the first 50 positive odd integers is 50^2. The sum of the first 50 positive even integers is

(**A**) 50^2 (**B**) 50^2+1 (**C**) 50^2+25 (**D**) 50^2+50 (**E**) 50^2+100

17. During 1996, the population of Sudbury decreased by 6% while the population of Victoria increased by 14%. At the end of the 1996, the populations of these cities were equal. What was the ratio of the population of Sudbury to the population of Victoria at the beginning of 1996?

(**A**) 47:57 (**B**) 57:47 (**C**) 53:43 (**D**) 53:57 (**E**) 43:47

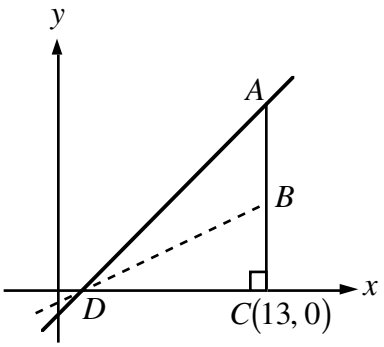
18. Given $A=\{1,2,3,5,8,13,21,34,55\}$, how many of the numbers between 3 and 89 cannot be written as the sum of two elements of the set?

(**A**) 43 (**B**) 36 (**C**) 34 (**D**) 55 (**E**) 51

19. In the diagram, the equation of line AD is $y=\sqrt{3}(x-1)$. BD bisects $\angle ADC$. If the coordinates of B are (p,q), what is the value of q?

(**A**) 6 (**B**) 6.5 (**C**) $\frac{10}{\sqrt{3}}$

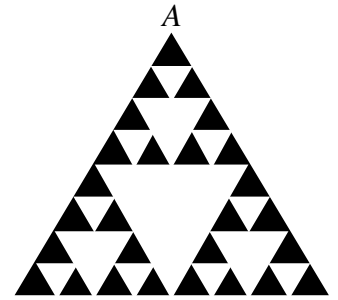
(**D**) $\frac{12}{\sqrt{3}}$ (**E**) $\frac{13}{\sqrt{3}}$



20. In the diagram, all triangles are equilateral. If $AB=16$, then the total area of all the black triangles is

(**A**) $37\sqrt{3}$ (**B**) $32\sqrt{3}$ (**C**) $27\sqrt{3}$

(**D**) $64\sqrt{3}$ (**E**) $\frac{64}{3}\sqrt{3}$



"Sierpinski Gasket"

Part C: Each question is worth 8 credits.

21. If $\dfrac{\left(\frac{a}{c}+\frac{a}{b}+1\right)}{\left(\frac{b}{a}+\frac{b}{c}+1\right)}=11$, and a, b, and c are positive integers, then the number of ordered triples (a,b,c), such that $a+2b+c\le 40$, is

(**A**) 33 (**B**) 37 (**C**) 40 (**D**) 42 (**E**) 45

22. If $2x^2 - 2xy + y^2 = 289$, where x and y are integers and $x \geq 0$, the number of different ordered pairs (x, y) which satisfy this equation is

(**A**) 8 (**B**) 7 (**C**) 5 (**D**) 4 (**E**) 3

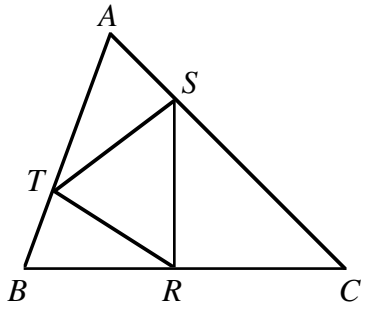
23. If $f(x) = px + q$ and $f(f(f(x))) = 8x + 21$, and if p and q are real numbers, then $p + q$ equals

(**A**) 2 (**B**) 3 (**C**) 5 (**D**) 7 (**E**) 11

24. The first term in a sequence of numbers is $t_1 = 5$. Succeeding terms are defined by the statement $t_n - t_{n-1} = 2n + 3$ for $n \geq 2$. The value of t_{50} is

(**A**) 2700 (**B**) 2702 (**C**) 2698 (**D**) 2704 (**E**) 2706

25. In triangle ABC, R is the mid-point of BC, $CS = 3SA$, and $\frac{AT}{TB} = \frac{p}{q}$. If w is the area of ΔCRS, x is the area of ΔRBT, z is the area of ΔATS, and $x^2 = wz$, then the value of $\frac{p}{q}$ is



(**A**) $\frac{\sqrt{21}-3}{2}$ (**B**) $\frac{\sqrt{21}+3}{2}$ (**C**) $\frac{\sqrt{21}-3}{6}$

(**D**) $\frac{\sqrt{105}+3}{6}$ (**E**) $\frac{\sqrt{105}-3}{6}$