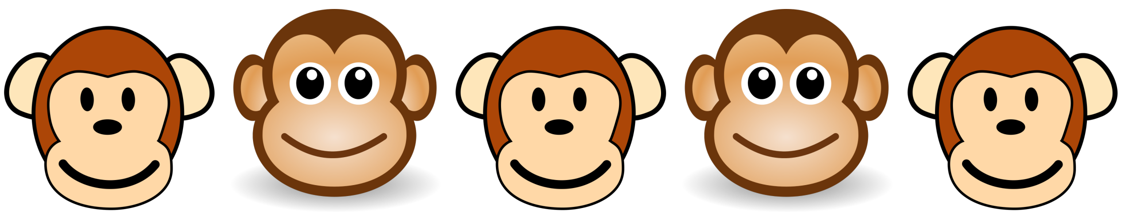
Problem of the Week
Grade 11 and 12

Five Prime Mates

The product of five different **odd** prime numbers is a five-digit number of the form $strst$, where $r = 0$.

Determine all possible numbers.



See the next page for a summary of divisibility tests for the integers 2 to 12.

Divisibility Tests

Divisibility by 2: A number is divisible by 2 if the last digit is even.

Divisibility by 3: A number is divisible by 3 if the sum of the digits is divisible by 3. For example, 1295 is not divisible by 3 since $1+2+9+5=17$ which is not divisible by 3. However, 1296 is divisible by 3 since $1+2+9+6=18$ which is divisible by 3.

Divisibility by 4: A number is divisible by 4 if the last two digits are divisible by 4. For example, 1295 is not divisible by 4 since 95 is not divisible by 4. However, 1296 is divisible by 4 since 96 is divisible by 4.

Divisibility by 5: A number is divisible by 5 if the last digit is a 0 or 5.

Divisibility by 6: A number is divisible by 6 if it is divisible by both 2 and 3. The number 395 is not divisible by 6 since it is not even and hence is not divisible by 2. The number 862 is not divisible by 6 since it is not divisible by 3 ($8+6+2=16$ which is not divisible by 3). The number 2964 is divisible by 6. It is even and is therefore divisible by 2. It is divisible by 3 since $2+9+6+4=21$ which is divisible by 3. Since 2964 is divisible by both 2 and 3, it is divisible by 6.

Divisibility by 7: We can follow an unusual algorithm to determine if an number is divisible by 7: Remove the unit's digit, double that digit and subtract it from the leftover number. If the difference is divisible by 7, the original number is divisible by seven. If unsure, repeat the algorithm with the new number.

Is 1356 divisible by 7? Remove the 6, double the 6 to 12, subtract from 135 leaving 123. Is 123 divisible by 7? Remove the 3, double the 3 to 6, subtract from 12 leaving 6. 6 is not divisible by 7 and therefore 1356 is not divisible by 7.

Is 45 024 divisible by 7? Remove the 4, double to 8, subtract from 4502 giving 4494. Repeat. Remove the 4, double to 8, subtract from 449 giving 441. Repeat. Remove the 1, double to 2, subtract from 44 giving 42 which is divisible by 7. Therefore, 45 024 is divisible by 7.

Divisibility by 8: A number is divisible by 8 if the last three digits are divisible by 8. For example, 1295 is not divisible by 8 since 295 is not divisible by 8. However, 1296 is divisible by 8 since 296 is divisible by 8.

Divisibility by 9: A number is divisible by 9 if the sum of the digits is divisible by 9. For example, 1295 is not divisible by 9 since $1+2+9+5=17$ which is not divisible by 9. However, 1296 is divisible by 9 since $1+2+9+6=18$ which is divisible by 9.

Divisibility by 10: A number is divisible by 10 if the last digit is a 0.

Divisibility by 11: We can follow an unusual algorithm to determine if an number is divisible by 11: Add the numbers in the even positions. Add the numbers in the odd positions. Subtract the two sums. If this difference is divisible by 11, the original number is divisible by 11.

Is 1 235 862 divisible by 11? The sums are $1+3+8+2=14$ and $2+5+6=13$. The difference of the sums is 1, which is not divisible by 11. Therefore, the number 1 235 862 is not divisible by 11.

Is 4 151 617 151 divisible by 11? The sums are $1+1+1+1+1=5$ and $4+5+6+7+5=27$. The difference of the sums is -22, which is divisible by 11. Therefore, the number 4 151 617 151 is divisible by 11.

Is 7 326 495 divisible by 11? The sums are $7+2+4+5=18$ and $3+6+9=18$. The difference of the sums is 0, which is divisible by 11. Therefore, the number 7 326 495 is divisible by 11.

Divisibility by 12: A number is divisible by 12 if it is divisible by 4 and 3. The number 394 is not divisible by 12 since 94 is not divisible by 4. The number 964 is not divisible by 12 since it is not divisible by 3. (The sum of the digits is 19 which is not divisible by 3.) The number 2964 is divisible by 12. The last two digits, 64, are divisible by 4 and therefore 2964 is divisible by 4. The sum of the digits is 21 which is divisible by 3. Since 2964 is divisible by 4 and 3, it is divisible by 12.