

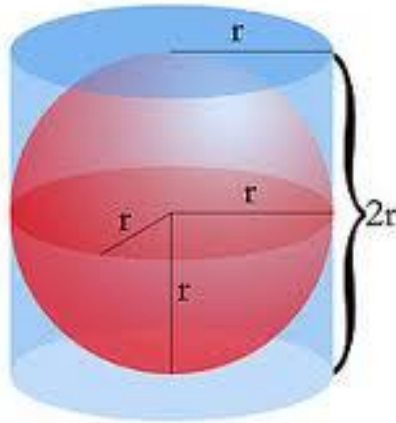
Surface Area and Volume of a Sphere

The formula for the surface area of a cylinder is composed of 2 circles as the base areas and a rectangle as the lateral area:

$$2\pi r^2 + 2\pi rh$$

If we placed a sphere inside of a cylinder so that it fit exactly, we would notice:

1. The cylinder has the same diameter as the sphere
2. The height of the cylinder is equal to the diameter ($d = 2r$) of the sphere



If the lateral surface area of the cylinder was made of paper, it could be cut and pasted on the surface of the sphere to cover it.

The length of the lateral surface is equal to the circumference of the sphere, $2\pi r$, and the height is equal to the diameter of the sphere, or $2r$.

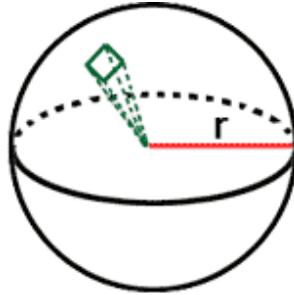
So, we multiply the length by the height to find the lateral surface area:

$$A_L = (2\pi r)(2r)$$

$$A_L = 4\pi r^2$$

So, this is also the formula for the surface area of a sphere with radius r .

Visualize a sphere covered with very small congruent squares, and each square is joined to the centre of the sphere to form a square pyramid. It would look something like this:



The volume of the sphere is actually just the sum of the volumes of these square pyramids!

$V =$ sum of volumes of all square pyramids

$V =$ sum of all the $[\frac{1}{3}(\text{base area})(\text{height})]$

But the height is just the radius and the sum of all the base areas is just the surface area of a sphere!

$V = \frac{1}{3}(\text{sum of all the base areas})(r)$

$V = \frac{1}{3}(4\pi r^2)(r)$

$V = \frac{4}{3}\pi r^3$

What About Hemispheres?

A **hemisphere** is half of a sphere. We can find the surface area and volume of hemispheres using the following formulas:

SA of a hemisphere = SA of one-half a sphere + area of a circle

$$SA = \frac{1}{2}(4\pi r^2) + \pi r^2$$

$$\mathbf{SA = 3\pi r^2}$$

V of a hemisphere = V of one-half a sphere

$$V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$$

$$\mathbf{V = \frac{2}{3}\pi r^3}$$

Example 1: Finding the Surface Area of a Sphere and Hemisphere

A globe has a diameter of 26 cm. What is the surface area of the sphere? Of each hemisphere?

Example 2: Finding the Volume of a Sphere and Hemisphere

The surface area of a sphere is 2560-cm^2 . What is the volume of the sphere? Of each hemisphere?