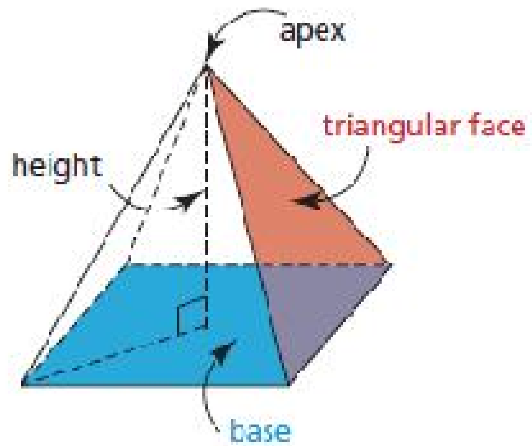
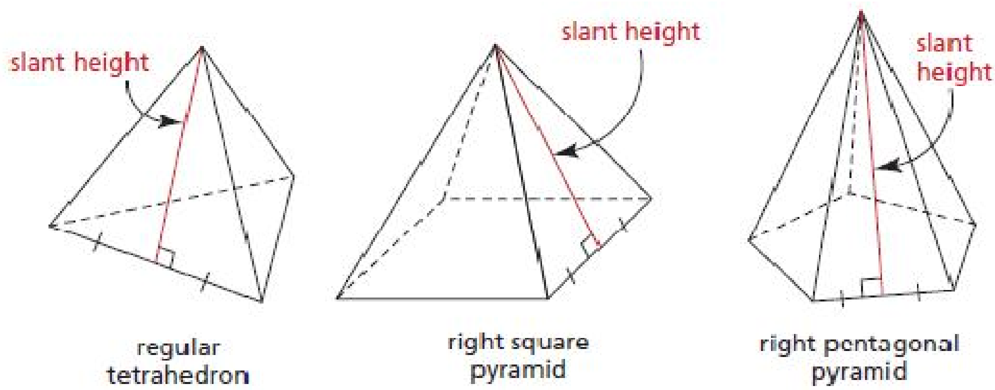
Surface Area of Right Pyramids and Right Cones

A **right pyramid** is a 3-dimensional object that has triangular faces and a base that is a polygon. The triangular faces meet at a point called the **apex**. The **height** of the pyramid is the perpendicular distance from the apex to the centre of the base.



When the base of a right pyramid is a regular polygon, the triangular faces are congruent. The shape of the base determines the name of the pyramid. The **slant height** of the right pyramid is the height of a triangular face.

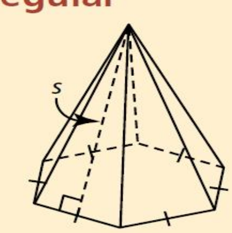


The surface area of a right pyramid is the sum of the areas of the triangular faces, called the **lateral area**, and the base.

Surface Area of a Right Pyramid with a Regular Polygon Base

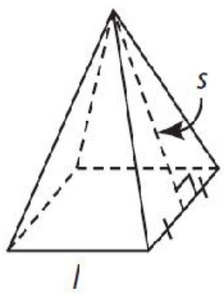
For a right pyramid with a regular polygon base and slant height s,

$$\text{Surface area} = \frac{1}{2}s(\text{perimeter of base}) + (\text{base area})$$



Want Proof?

We can determine a formula for the surface area of any right pyramid with a regular polygon base. Consider this right square pyramid. Each triangular face has base ℓ and height s.



The area, A, of each triangular face is:

$$A = \frac{1}{2}\text{(base)(height)}$$

$$A = \frac{1}{2}\ell s$$

So, the area of the 4 triangular faces is:

$$4\left(\frac{1}{2}\ell s\right) = 4\left(\frac{1}{2}\right)\ell s$$ Rearrange the numbers and variables.

$$= \left(\frac{1}{2}s\right)(4\ell)$$

The area of the triangular faces of a pyramid is called the **lateral area,** denoted A_L.

$$A_L = \left(\frac{1}{2}s\right)(4\ell)$$

The term 4ℓ is the perimeter of the base of the pyramid, so

Surface area of the pyramid $= (\frac{1}{2}$ slant height)(perimeter of base) $+$ base area

Since any right pyramid with a regular polygon base has congruent triangular faces, the same formula is true for any of these pyramids.

Example 1: Finding the Surface Area of a Square Pyramid

A square pyramid has a base with side length 4 ft. Each triangular face has a slant height of 5 ft. Find the surface area of the pyramid.

Example 2: Finding the Surface Area of a Regular Tetrahedron

A regular tetrahedron has a base with side length 7 cm. Each triangular face has a slant height of 6 cm. Find the surface area of the tetrahedron.

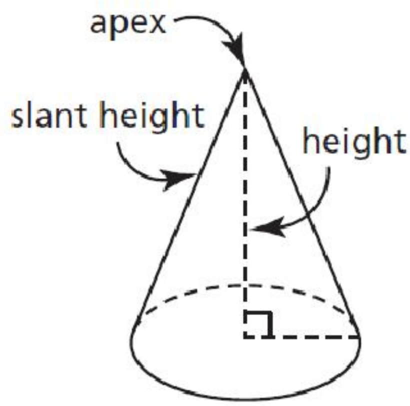
Example 3: Finding the Surface Area of a Rectangular Pyramid

A rectangular pyramid has two sets of congruent triangular faces: one has a base of 5-m and a height or 6-m; the other has a base 9-m and a height of 5-m. Find the surface area of the rectangular pyramid.

Example 4: Finding the Surface Area of a Square Pyramid Given Its Height

Find the surface area of a square pyramid with a base of 10 in. and a height of 6 in.

A right circular cone, or **right cone**, is a 3-dimensional object that has a circular base and a curved surface. The **height** of the cone is the perpendicular distance from the **apex** to the base. The **slant height** of the cone is the shortest distance on the curved surface between the apex and a point on the circumference of the base.

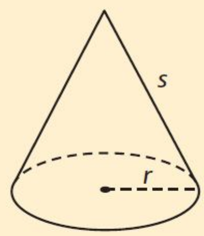


Surface Area of a Right Cone

Surface area = lateral area + base area

For a right cone with slant height s and base radius r:

$SA = \pi rs + \pi r^2$



Want Proof?

Hint: Start with a right square pyramid and then continue adding more sides to the polygon base. As the number of sides increases, the base approaches the shape of a circle, and the perimeter will become the circumference of a circle. Try it for yourself!

Example 5: Finding the Surface Area of a Cone

A right cone has a base radius of 12 ft. and a height of 14 ft. Calculate the surface area of the cone to the nearest square foot.

Example 6: Determining an Unknown Measurement

The lateral area of a cone is 420 cm^2. The diameter of the cone is 15 cm. Determine the height of the cone to the nearest tenth of a centimetre.